PHD

The Role of Information and Liquidity in Interbank Network Formation

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Di Xiao

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University of Bath

Department of Economics

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Abstract

Interbank markets are critical to banking systems, as they enable banks to manage liquidity and allow monetary policy to go through. As the 2007 global financial crisis shows, an unexpected "freeze up" of interbank markets can cause difficulties for central bank interventions and the overall economy, which demonstrates the need to better understand systemic risk. To this aim, a comprehensive picture of interbank markets and their lending networks is necessary and of much importance. The objective of this study is to deepen our understanding of interbank network formation, which has received relatively less attention. We hope to better perceive how factors like asymmetric information and liquidity preference influence the interbank structure, and to further explore impacts from monetary policy.

In the first essay of this study, we develop a model of the formation of interbank lending network, with risk aversive banks maximising expected profits in the case of information asymmetry. The equilibrium interbank structures are investigated to understand how a "freeze" market is likely to occur in a stress situation. In the second paper of this study, we introduce another interbank network formation model where a large number of homogenous banks having preferences to both profits and cash. Interest rates are determined endogenously through an internal trade off between these preferences. Realistic network structures are found in the interbank market of computer experiments. The third essay further considers the impact of monetary policy on the interbank market proposed in essay 2. The key features of the interbank market structure remain roughly unchanged after central bank operation. However, banks of different network positions are not symmetrically impacted.

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Chapter 1

Introduction

Contents

1 Motivation and research objective

Interbank markets are critical to banking systems because they enable banks to manage, exchange and redistribute liquidity and allow monetary policy to be implemented and transmitted to the wider economy. The well functioning of the interbank markets are thus crucial to the health of the overall economy. During the 2007 global financial crisis, many interbank markets experienced severe liquidity shortages. Banks were reluctant to rollover short-term loans, making interbank borrowing rates to rise record high. Moreover, this "freeze up" of interbank markets caused difficulties for central bank interventions that aimed to provide liquidity to banks and led to a resort to unconventional monetary policies. Compared to the fact that the failure of sub prime mortgage market was actually predicted by some people, the liquidity freeze was much less anticipated, suggesting room to improve the understanding of interbank markets. To achieve this aim, in the post crisis literature, numerous efforts embracing different perspectives have been made in relevant fields of study.

One burgeoning area is to represent interbank borrowing/lending as a network, i. e. each bank is represented by a node and an interbank loan is denoted by a directed linkage. This network perspective is very useful in analysis of a financial contagion, where a (solvency or liquidity) problem of one single bank can be spread along a chain of banks through interbank market, bring down multiple banks along the way, and may eventually endanger the entire banking system (Allen & Gale (2000), Martínez-Jaramillo et al. (2010), Gai et al. (2011)). Financial contagion is an important form of systemic risk that is disruptive to the stability of financial system, as evident in 2007 financial crisis. There are two important aspects that need to be considered in the study of financial contagion through interbank network, yet received relatively less attention. One is that the network is not likely to be static because banks are able to react to arrivals of various information and adjust their behaviours accordingly. In other words, the interbank network may change drastically in response to a possibility of crisis. Therefore, it is meaningful to study how network can be formed by strategic behaviours of banks. The other aspect is that interbank network structure matters for the spread of financial contagion, which is shown by substantial studies (Allen & Gale (2000), Gai & Kapadia (2010), Acemoglu et al. (2015)). In addition, as demonstrated by empirical studies, interbank networks in reality are found to possess a number of topology features including a core periphery structure that arise endogenously (Craig & Von Peter (2014), Langfield et al. (2014), Fricke & Lux (2015)). To understand the interbank market properly, it is crucial to include these realistic features in modeling the formation of interbank network.

The aim of this thesis is to contribute to the topic of interbank network formation. Studies on the network formation of interbank markets are relatively few among the overall interbank network literature. The mechanism underlying the formation, the factors affecting the network and the emergence of certain structures of the interbank market are not thoroughly understood. We thus are interested in contributing to the literature by studying how interbank networks are formed and using network formation models to shed light on the interbank borrowing/lending and monetary policy. Essay 1 presents a model of interbank network formation that allows us to study the equilibrium network structures, where banks assess counterparty default risks based on asymmetric information and seek to maximize profits from interbank lending. Essay 2 introduces another interbank network formation model where a large number of homogenous banks under random reserve shocks favour both profitability and liquidity. Different from essay 1, the counterparty default risk is neglected and there is no asymmetric information in essay 2. Essay 3 is built on the model of essay 2 and introduces a central bank operation before the commencement of the interbank market. How liquidity injection/extraction effects interbank market is studied, especially in terms of interbank network properties. Essay 1 and 2 compliments each other as they deal with the network formation of different interbank markets where banks have different primary concerns. In essay 1, banks are risk averse, concerned with borrowers' insolvency and faced with asymmetric information. This applies to the interbank market in a stress situation. In essay 2, on the contrary, interbank lending is assumed to be very safe and banks think more cash is desirable, which is similar to a precautionary need. This applies to a non-stress interbank market that is believed to be close to risk free. Both fear for counterpaty insolvency and precautionary need for liquidity are possible explanations as to why the interbank "freeze" occurred in financial crisis. Essay 1 and 2 also differ in determination of interbank lending rates. Interest rates in essay 1 are exogenously given while they are endogenously determined in essay 2 and followed by essay 3.

2 Summary of essays and contributions

This section summarises the contents of three essays. Briefly, in essay 1, we propose a model to study the short-term interbank lending from a network formation perspective, find equilibrium structures of the interbank network and show how their occurrences depend on banks' individual behaviours. In essay 2, we develop a model of the interbank lending based on liquidity and profitability considerations of homogeneous banks and find the established network having realistic features. In essay 3, we further introduce a central bank operation in form of liquidity auction before the interbank market based on essay 2, show the equilibrium outcome of the auction under different tenders and assumptions, and analyse how the interbank network structure and interbank rates are influenced comparing to a benchmark of no such operation.

2.1 Essay 1

We develop a model of network formation to study the short-term interbank lending. Banks have deep pockets, i. e. they are able to lend and borrow without any constraint. Each bank is a borrower and/or lender. A lender assesses each borrower individually by a utility function that increases when expected return increases or variance of the return decreases. The possibility of a borrower's insolvency is embedded in the uncertainty of return. The banks first receive information containing a public and a private (noise) signal about the solvency of other banks. We assume banks to be homogenous, i. e. all banks have identical expected returns and only their private signals are different. They may then observe each other's lending decisions. The existence/absense of lending generates information on the lender's private signal. For instance, if the lending exist, the lender must expect a high enough return that compensates its exposure to risk. Other banks knowing the public signal can infer some knowledge about this lender's private signal and adjust their assessment to the borrower accordingly.

Equilibrium structures of a three-bank interbank network is determined and evaluated. We first analyse individual lending strategies and obtain the probability of choosing each strategy. We then examine strategy combinations and apply an algorithm to determine all possible equilibria and their corresponding probabilities. There are 16 possible equilibrium interbank network structures in total. Among them, four are dominant in terms of occurrence. Banks in these four equilibrium structures are observed to exhibit herding behaviour, i. e. they either lend to the same borrower or refuse to do so simultaneously. We find that how likely each structure is to occur depends on banks' risk aversion, the expected returns on interbank loans, and uncertainty of information. We analyse the effect of each factor by holding others constant. The equilibrium structures help to shed some light on the liquidity freeze in the interbank market. In a stressed market featuring high expectations of insolvency, banks eventually form a frozen market where no bank is willing to lend to any other bank. In a stressed market featuring high risk aversion, banks may have multiple equilibrium structures including a freeze, because banks may either refuse lending or follow each other's decisions. We then consider heterogenous banks and find it not affecting the outcome substantially. The results obtained for the homogeneous case can thus be shown to be robust.

2.2 Essay 2

In this essay, we introduce a model of interbank network formation in a large homogeneous banking system. The interbank market considered here can be thought as an overnight market where interbank loans are thought to be of little default risk. Banks are assumed to neglect default risk and focus instead on liquidity management and profitability. To be more precise, banks are assumed to have preferences towards higher cash reserve (liquidity) and higher return on equity (profitability) in a utility function of Cobb-Douglas form. The balance sheet of a bank is also specified with all long-term items and rates fixed. Banks are all alike in terms of leverage, liquidity holdings and size at the beginning, but suffer from exogenous and random liquidity shocks. To model behaviours of individual banks, or how they ask/offer interbank loan rates specifically, we borrow the idea of the inventory model of market makers in market microstructure literature. Banks consider the optimal amount of liquidity to hold (their "inventory") and are willing to deviate from it by adjusting the price (interest rate) of the asset (interbank loan) to increase their overall utility, see Stoll (1978) for the original model. Providing a loan to another bank will reduce liquidity and at the same time increase returns due to the interest earned, while obtaining such a loan will increase liquidity and reduce returns as interest is being paid.

We study some key properties of the banks' individual behaviours. We show that a bank has an optimal liquidity ratio conditional on an interbank market with no arbitrage opportunities and that a bank does not seek a loan from a borrower in a subsequent transaction, consistent with reality. Most importantly, we calculate, for a given liquidity and return level, the reservation rate at which each bank is willing to borrow and lend to other banks. Here we interpret banks as resembling market makers that are willing to "buy" and "sell" liquidity. The spread of a bank's reservation rates between lending and borrowing is found to be always positive. The effects of changing liquidity and the return on a bank's reservation rates and spread are also derived.

Banks may trade any excess liquidity in the interbank market by lending to banks with liquidity shortfalls in order to maximize utility, so that reserves are reallocated among banks. The trading is organized through a decentralized (over the counter) market, where banks meet bilaterally for lending/borrowing with standardized size of each transaction. A bank ask other banks in a random order for quotes and make transactions when the quotes are more favourable than its reservation prices. The bank also remembers who it has asked for quotes and can have multiple trades with the same bank if its offer is still better than that of any other bank. When all banks have been asked, another bank is selected randomly to continue the process until no further transaction is possible among all banks.

In the evaluation of results, we run a large number of computer experiments with random parameter setting. We first focus on the properties of the obtained interbank networks. The vast majority of these interbank networks exhibit a core-periphery structure. Besides, other realistic properties that are commonly found in empirical studies are also observed, such as low density, negative assortativity, fat tails of the degree distribution and short average path lengths. We then analyse how the emergence of the core-periphery structure is affected by parameter values. We find the core-periphery structure is more pronounced when banks are less concerned about liquidity but more about returns, or when they have higher leverage. A larger banking system or a larger size of each interbank loan also contribute to the emergence of core-periphery structure, while reducing the size of core. We then consider the properties of banks in the core and periphery. Banks in the core are borrowing at lower rates than those in the periphery and are lending at higher rates. They are also much more active, with higher interbank borrowing and lending, higher leverage, but thinner cash. As a result, they enjoy higher return and utility compared to banks in periphery. We also analyse determinants of a bank's position in interbank network and find that banks that enter the market early or face a larger liquidity shock are most likely to become core banks.

2.3 Essay 3

In this essay, we introduce a central bank into the banking system modeled in essay 2. The central bank conducts monetary policy through an auction of short-term central bank funds. The central bank may choose to implement either liquidity injection or liquidity extraction via the auction. The auction is held before the start of the interbank market after banks receiving idiosyncratic liquidity shocks. As a result, banks participating in the auction have varying liquidity levels and thus heterogenous valuations for liquidity, which we assume to be public information. They submit bid schedules strategically to maximize the overall utility. The utility function used by banks is identical to that in essay 2. After the auction (central bank lending/borrowing), banks' liquidity positions and returns are updated and they can further adjust their liquidity holdings through the interbank market.

We first view the central bank operation and the interbank market for reserves as separated, i. e. banks do not consider funding through the interbank market when bidding for central bank funds. We consider two types of tenders for the central bank operation, a fixed rate operation with full allotment and a variable rate operation. For the same banking system, a bank are shown to have different individual valuation and bid schedule when we switch the type of operation. Specifically, the bid schedule in the variable rate tender has a flat area as opposed to that in the fixed rate tender with a downward sloping shape. In both tenders, bid shading is present though not of the same degree. However, the above differences do not affect the equilibrium price of central bank funds as the same equilibrium outcome can be achieved through either type of tender.

We then view the central bank operation and the interbank market for reserves as interdependent. Banks form some anticipation of the cost of funding later through the interbank market when they bid for central bank funds. And banks here seek to maximize the overall utility from the borrowing/lending from/to both the central bank and the interbank market, instead of only considering the utility gain from central bank operation. This not only alters the equilibrium rates for central bank funds but also influences subsequent interbank trading. We also run a large number of computer experiments with randomized parameter values and find the equilibrium rate in central bank operation to be also dependent on other determinants.

We further analyse the effects of monetary policy by comparing each of the settings above with the benchmark case of no central bank operation. We find the basic structure of the interbank network roughly maintains, with or without the central bank operation, i. e. core-periphery structure, small core, low-density network. However, liquidity injection does make the core-periphery structure less pronounced while liquidity extraction has the opposite effect. Also, both liquidity injection and extraction reduce the number of active banks and amount of trading in the interbank market. Core/periphery banks are found to still lend at higher/lower rates in the interbank market after the central bank operation. However, we find core and periphery banks are affected differently by central bank operation, in terms of exposure to interbank market and reliance on central bank funds etc. In addition, periphery banks are found to be under stronger influence than core banks.

2.4 Contributions

The main contributions of this thesis is to help to the understanding of the formation of interbank network in multiple ways.

Essay 1 contributes to the understanding of the liquidity freeze from a network perspective. It provides insights to show the presence of asymmetric information gives rise to a freeze in interbank lending. Especially, we show the interbank network features multiple equilibrium structures, i. e. in a stress situation both freeze and non-freeze market structures are possible. This contributes to the understanding of the fragility of the financial market, being possible to flip from a well-functioning one to a mal-functioning one.

Essay 2 helps to understand the emergence of network structure from a framework where banks have preferences for not only profits but also liquidity. Interest rates are determined based on the tradeoff between these preferences. It also contributes to the literature on the interbank network formation by analysing what drives the interbank lending networks to possess realistic features in terms of network structure and endogenous interest rates.

Essay 3 how the presence of a central bank may affect the interbank network through liquidity injection/extraction. It contributes to the understanding of how monetary policy goes through the banking system by showing that key properties of the interbank market more or less remains, yet banks of different positions are asymmetrically impacted by monetary policy.

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Chapter 2

Essay 1 - Equilibrium Interbank Lending Networks

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Contents

Equilibrium interbank lending networks

Di Xiao* Andreas Krause†

Abstract

We propose a model to study short-term interbank lending from a network formation perspective. Banks, being provided with public and private signals about the solvency of other banks, decide on interbank lending by also considering the decision of other banks to lend. We observe that the dominant equilibrium networks are those where banks follow each others' decisions, making the equilibria very vulnerable to shifts in expectations. The networks range from fully connected (highly liquid markets) to empty networks (frozen markets) and we derive the conditions under which they emerge.

1 Introduction

As evidenced, for example, during the 2007 global financial turmoil, a characteristic of a financial crisis is contagion where a relatively small event, like the failure of a single financial institution, may trigger a chain reaction. This failure can spread to the whole financial system and eventually reach out to the real economy. In order to capture the connections between financial institutions a network approach has been chosen that focuses on interbank lending.

Financial systems can be seen as networks, with nodes representing individual financial institutions, and links representing their bilateral exposures, such as interbank loans, credit lines or derivatives positions. Starting from the parsimonious financial network of Allen & Gale (2000) which consists of only four banks, scholars have

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investigated various financial networks for a wide range of markets that are increasingly detailed and realistic, see Upper (2011) for a review of interbank network properties. The network structure is found to be greatly influential for the systemic risk of a banking system, where the failure of a single bank may cause the failure of a significant number of other banks and endanger the stability of the whole banking system (see e.g. Nier et al. (2007), Gai & Kapadia (2010), Acemoglu et al. (2013)). Although some universal features of financial networks have been discovered and studied, many questions await further exploration, such as how these features arise from interactions between individual financial institutions, how they further influence other banks' behaviors, and how these interactions may help to escalate or mitigate systemic risk.

One of the motives behind this work is to offer some insights into the phenomenon of liquidity freezes in interbank markets as was clearly observed and highly noted in the 2007 financial crisis. Money markets that were used to be highly liquid suddenly saw a "freeze" in liquidity, with extremely high borrowing rates. Two possible explanations are offered as to why market players with excess liquidity are not willing to lend to the market in Gale & Yorulmazer (2013): fear of counterparty risks and liquidity hoarding (fear of a future liquidity shortage). These two fears may intertwine together to cause a "freeze" in interbank lending. Among the literature referring to financial network formations, few have focused on explaining liquidity freezes, yet it should be interesting to explore this issue from the network formation perspective and how such freezes might emerge.

In this paper, based on the idea in Fique & Page (2013) and developed further, we propose a model for the formation of short-term interbank loan networks, under the assumption that banks can observe each others' lending decisions and adjust their evaluations of borrowers accordingly. Specifically, this assumption allows banks to exchange views about whether to lend to a borrower without fully revealing their private signals. Under this assumption, a bank's lending decision generates information to other banks and they may sometimes exhibit herding in making lending decisions. Our primary interest is to find equilibrium structures of interbank networks ranging from a complete network (fully liquid market) to an empty network (market freeze), and show how their occurrence depends on individual behaviour. We find that how likely each structure is to occur depends on banks' risk aversion, the expected returns on interbank loans, and uncertainty of private information. For a stressed market, featuring low expectations of debt paying ability, banks eventually cause a frozen market. For a stressed market featuring high risk aversion, banks may have multiple equilibrium structures including a freeze, because banks may either refuse lending or follow each other's decisions.

The main part of this article is organized as follows: in section 2, we review the related literature and section 3 develops the model of how individual banks form and update their beliefs while section 4 assesses their decisions about interbank lending and describes the algorithm used to determine the equilibrium network structures. We analyse the probability distribution of equilibrium networks in section 5 and section 6 discusses some policy implications before we conclude our article in section 7.

2 Determinants of interbank lending

Firstly, this work is related to a strand of literature investigating systemic risk in interbank markets. The mechanisms of contagion can be generally classified into two types: contagion through direct links, which refers to a situation where one insolvent bank may cause losses to its creditor bank and consequently triggers insolvency among its creditors; and secondly contagion through indirect links, which includes liquidity hoarding, common assets exposure under fire sales, and bank runs.

Studies of contagion through direct links are perhaps most abundant. Some important early models all follow the standard model of Diamond & Dybvig (1983) but extend it with a network perspective. However, these models consider only very simple network structures, usually only three structures are studied, a complete network, a ring network and an empty network. Allen & Gale (2000) and Freixas et al. (2000) are two such early standard models, which have much similar settings. Banks have incentives to hold bilateral exposures, changing deposits (Allen & Gale (2000)) or having credit lines (Freixas et al. (2000)), due to uncertainty about deposit withdrawals, uncertainty of when depositors they consume (Allen & Gale (2000)) or where they consume (Freixas et al. (2000)). Then under the three different network structures, they examine how the shocks spread out through a liquidity preference shock (Allen & Gale (2000)) or risky long-term assets (Freixas et al. (2000)). Similar conclusions are drawn from both models, a complete network is usually more stable than an incomplete network (a ring network) since there are more banks to share the loss of a given shock. Following this strand of literature, Babus (2007), Leitner (2005), and Allen et al. (2010) investigate network formations in which banks choose to form a network that maximize their utility through risk diversification, profit maximization or reducing the risk of contagion. Another strand of literature develops models of financial contagion based on more complex networks, mainly random graphs, and most of these studies have noted that some sort of "phase transition" of systemic risk happens in such networks, which is also referred to by Haldane (2013) as financial networks having "robust-yet-fragile" feature. In models like Gai & Kapadia (2010) and Gai et al. (2011), this feature means that there exist tipping points for the level of connectivity, and that sharp changes in levels of systemic risk (the extent of contagion) occur around these tipping points. Acemoglu et al. (2013) find that tipping points also exist for the size of initial shock. Amini et al. (2012) find a necessary condition for contagion to not encompass the entire network and develop a resilience measure which is a function of each bank's connectivity and fraction of contagious links. All tipping points for macroeconomic shocks correspond to positive resilience measure values. Caccioli et al. (2012) develop the work of Gai & Kapadia (2010) by adding heterogeneity into the model and show that heterogenous connectivity, the size of banks and degree correlations play a role in determining the stability of a financial system.

Studies of contagion through indirect linkage include liquidity hoarding, common asset exposures under fire sales, and bank runs. Rochet & Vives (2004) present a model showing that banks can be illiquid but solvent. They find that in equilibrium it is possible that solvent banks still fail to obtain liquidity from interbank markets due to the liquidity hoarding of informed investors. For this reason, they argue the importance of a lender of last resort. Heider et al. (2015) present a model that highlight the role of asymmetric information in the assessment of counterparty risk leading to liquidity hoarding among liquid potential providers of funds. The result is that the market fails to reach the desirable equilibrium but instead reaches a liquidity freeze. Acharya & Skeie (2011) present a model showing precautionary liquidity hoarding arising from lenders fearing rollover risk that can help to explain market stress, where high rates and low volumes for borrowing are the result of high leverage and the illiquidity of assets. Gale & Yorulmazer (2013) present a similar model of banks' choices between liquid and illiquid assets in a portfolio where banks have incentives to hoard liquidity not only due to rollover risk but also have the opportunity to buy fire sale assets of other banks that face liquidity shortages. Finally, Brunnermeier & Pedersen (2009) present a model that considers the interplay of market liquidity and funding liquidity and shows that they can be mutually reinforcing as liquidity spirals when margins are destabilising.

Secondly, this work is also related to another growing strand of literature on network formation games as applied to financial systems. One strand sees the network formation as a static game, like Leitner (2005), Babus (2007), Acemoglu et al. (2013), where all individuals make decisions simultaneously. They are all based on the standard model of Allen & Gale (2000) but extend it to N banks and assume banks consider the risk of potential shocks to them or to their neighbors when choosing links. In Leitner (2005), this is done by a social planner solving for and then proposing an optimal network based on the banks' random initial endowments with banks choosing to accept the proposal or have an empty network. In making decisions, banks try to balance the tradeoff between risk sharing and the risk of contagion. This is because being linked in a network on the one hand offers resources to lend from when one is facing an unexpected liquidity shock, yet on the other hand, one may suffer from losses when others withdraw money. In the model of Babus (2007) banks are classified into two types, and banks play a network formation game within each type. The primary concern of a bank is to prevent the risk of contagion, thus a bank chooses the network where no single neighbor's liquidation should lead to its own bankruptcy. In the model of Acemoglu et al. (2013) banks propose debt contracts conditional on borrowers' lending behaviour. As a result Leitner (2005) find the network can be ex ante optimal, but a collapse of the whole system may still occur in some cases. Babus (2007) show the network can be resilient with a large number of banks, which means the probability of contagion can be close to zero. Acemoglu et al. (2013) find that the equilibrium interbank network formed can be vulnerable to contagion, due to the presence of financial network externalities which cause the emergence of socially inefficient network.

Another strand of this literature see the network formation as a process of evolution, e.g. Lenzu & Tedeschi (2012) and Anand et al. (2012). In the model of Lenzu & Tedeschi (2012) banks form a network from random rewiring. The possibility a bank links to another bank depends on the latter's profitability, a bank's in-degree is thus a signal for its profitability. How much banks value this signal influences the structure of the network eventually formed. In the model of Anand et al. (2012) rollover decisions are made as in a foreclosure game. Whether a bank chooses to rollover depends on its cost of miscoordination and the borrower's asset-to-liability ratio, which are random and time-dependent. Consequently, the network structure evolves overtime, and the average connectivity in the stationary state depends on debt maturity and miscoordination cost. There is also model like Cohen-Cole et al. (2010) that use a combination of static and dynamic games.

Among the work studying interbank network formation, we especially refer to Fique & Page (2013) where rollover decisions of banks depend on the existing architecture of interbank networks. We keep the salient feature of their model, which is a bank's lending decision provides information to other banks, but have notable differences in both the details of the approach taken and aim of the model.

3 A model of interbank lending decisions

We assess a banking system in which N banks are considering lending to each other. For simplicity we assume that all loans are of the same size and mature in a single time period such that we can focus on the rollover decisions by banks. Furthermore, there are no constraints on the ability of banks to lend to each other and banks are always willing to accept any loans they are given. This simplification allows us to concentrate on the lending decision itself rather than having to consider the impact of any such constraints on the outcomes. Banks only differ in their risk of repaying this loan and the signals banks receive about this risk.

Interbank lending can be represented by a directed network where each node represents a bank and the edges the existence of a loan between two banks. This network can be represented by an adjacency matrix $A = \{a_{ij}\}_{i,j=1,\dots,N}$, where $a_{ij} = 1$ if bank i lends to bank j and zero otherwise. Obviously we require that $\forall i = 1, \dots, N : a_{ii} = 0$.

Decisions on interbank lending are assumed to be done decentrally such that each loan is assessed individually by a risk averse decision-maker maximizing expected utility. Approximating the expected utility of bank i giving a loan to bank j in the usual way with absolute risk aversion $\lambda_i \geq 0$, we get

$$U_{ij} = a_{ij}\mu_{ij} - \frac{1}{2}\lambda_i a_{ij}\sigma_{ij}^2, \tag{1}$$

where μ_{ij} and σ_{ij}^2 denote the expected value and variance of the return bank i believes to be generated from lending to bank j. This return will not only include the interest charged, but most importantly also include the possible default of bank j and the subsequent losses arising from this.

The following section will now explore how these expected values and variances are determined. We will use information based on public and private signals in a first step to assess each loan and then in addition also use information based on the lending decisions of other banks.

3.1 Assessment of private signals on interbank lending returns

Banks know that the true return of bank i, r_i, is a random variable that is normally distributed with mean μ_i and common variance σ^2:

$$r_i \sim N\left(\mu_i, \sigma^2\right). \tag{2}$$

This true return, however, cannot be directly observed. Instead each bank receives a private noisy signal that is independent of the true return as well as independent across banks:

$$\begin{aligned} \widehat{r}_{ij} &= r_j + \varepsilon_{ij}, \\ \varepsilon_{ij} &\sim N\left(0, \sigma_\varepsilon^2\right). \end{aligned} \tag{3}$$

Here bank i is lending to bank j. A bank can then use the noisy signal $\widehat{r}_{ij}$ to infer the true return of the borrower using conditional expectations:

$$\begin{aligned} \mu_{j|i} &= E\left[r_j|\widehat{r}_{ij}\right] = \mu_j + \frac{\sigma^2}{\sigma_r^2}\left(\widehat{r}_{ij} - \mu_j\right), \\ \sigma_{j|i}^2 &= Var\left[r_j|\widehat{r}_{ij}\right] = \frac{\left(\sigma_r^2 - \sigma^2\right)\sigma^2}{\sigma_r^2}, \end{aligned} \tag{4}$$

where $\sigma_r^2 = Var\left[\widehat{r}_{ij}\right] = \sigma^2 + \sigma_\varepsilon^2$. Hence the return bank i receives from lending to bank j, $r_{j|i}$, is believed to be a random variable distributed as follows:

$$r_{j|i} \sim N\left(\mu_{j|i}, \sigma_{j|i}^2\right). \tag{5}$$

Similarly we know that private signals for bank j will also affect our assessment of bank k's signal about bank j due to the correlation between the two signals:

$$\begin{aligned} \mu_{kj|ij} &= E\left[\widehat{r}_{kj}|\widehat{r}_{ij}\right] = \mu_j + \frac{\sigma^2}{\sigma_r^2}\left(\widehat{r}_{ij} - \mu_j\right), \\ \sigma_{kj|ij}^2 &= Var\left[\widehat{r}_{kj}|\widehat{r}_{ij}\right] = \frac{\sigma_r^4 - \sigma^4}{\sigma_r^2}, \end{aligned} \tag{6}$$

and hence

$$r_{kj|ij} \sim N\left(\mu_{kj|ij}, \sigma_{kj|ij}^2\right). \tag{7}$$

Finally we can easily verify following Johnson & Kotz (1970) that the correlation between these updated signals is given by

$$\rho_{j|i,kj|ij} = \frac{\sigma^2}{\sqrt{\sigma^2 + \sigma_r^2}}, \tag{8}$$

This correlation can now be used to determine the joint distribution of $r_{kj|ij}$ and $r_{j|i}$.

3.2 Updating beliefs from lending decisions

In addition to the private signal, banks can also extract information from the behavior of other banks, i. e. whether they lend or not to a specific bank. This decision will reveal partially the private signal the other bank has received and can be taken into account when assessing one's own lending decision. A bank will only lend if $U_{ij} \geq 0$ as for non-lending we have that due to $a_{ij} = 0$ it is $U_{ij} = 0$.

Once the information from the private signal has been assessed as in the previous section, banks will assess the information available from other banks' lending decisions separately. The following lemma provides the results of these considerations:

Lemma 1. *Observing the decision of another bank k to lend to bank j, the assessment of the expected return of bank j and its variance by bank i are given by*

$$\begin{aligned} \widehat{\mu}_{ij|k} &= E\left[r_{j|i} | U_{kj} > 0\right] \\ &= \mu_{j|i} + \rho_{j|i,kj|ij}\sigma_{j|i}\frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}, \\ \widehat{\sigma}_{ij|k} &= Var\left[r_{j|i} | U_{kj} > 0\right] \\ &= \sigma^2_{j|i}\left(1 - \rho_{j|i,kj|i}\frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} \times \right. \\ &\qquad \left.\left(\frac{\phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{1 - \Phi\left(\frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)} - \frac{\gamma - \mu_{kj|ij}}{\sigma_{kj|ij}}\right)\right), \end{aligned} \tag{9}$$

where $\gamma = \left(\frac{\sigma_r^2 - \sigma^2}{\sigma^2}\right)\left(\frac{1}{2}\lambda_i\sigma^2 - \mu_j\right)$. Similarly we can get those moments for the case *that bank k does not lend to bank j:*

$$\begin{aligned}
\widehat{\mu}_{ij|-k} &= E\left[r_{j|i}|U_{kj}<0\right] \\
&= \mu_{j|i} - \rho_{j|i,kj|ij}\sigma_{j|i}\frac{\phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)}, \\
\widehat{\sigma}_{ij|-k} &= Var\left[r_{j|i}|U_{kj}<0\right] \\
&= \sigma^2_{j|i}\left(1-\rho_{j|i,kj|i}\frac{\phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)}\times\right. \\
&\qquad \left.\left(\frac{\phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)}{\Phi\left(\frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)} - \frac{\gamma-\mu_{kj|ij}}{\sigma_{kj|ij}}\right)\right),
\end{aligned} \tag{10}$$

Proof. The proof is a straightforward application of the moments of truncated normal distributions, where we note from (1) that $U_{kj}>0$ is equivalent to $\widehat{r}_{kj}>\gamma$ if we replace μ_{ij} with $\mu_{j|i}$ and σ_{ij} with $\sigma_{j|i}$. □ □

Based on this lemma we can thus determine the expected returns and variance of bank j as assessed by bank i:

$$\begin{aligned}
\mu_{ij} &= \begin{cases} \widehat{\mu}_{ij|k} & \text{if} \quad a_{kj}=1 \\ \widehat{\mu}_{ij|-k} & \text{if} \quad a_{kj}=0 \end{cases}, \\
\sigma_{ij} &= \begin{cases} \widehat{\sigma}_{ij|k} & \text{if} \quad a_{kj}=1 \\ \widehat{\sigma}_{ij|-k} & \text{if} \quad a_{kj}=0 \end{cases}.
\end{aligned} \tag{11}$$

These expressions can now be inserted into equation (1) to assess the utility of bank i from lending to bank j. As this utility will depend on the behavior of another bank, k, we will rewrite this utility as $U_{ij}(a_{kj})$. The coming section will now discuss how the equilibrium in this model can be determined for the special case of $N=3$.

4 Determination of equilibria in a three-bank system

In the coming sections we will restrict our analysis to a banking system with three banks. While such a restriction seems unrealistic for most actual banking systems, it

	Conditions	Probability	Strategy
1	$U_{ij}(0) < 0$, $U_{ij}(1) < 0$	$\Phi(\frac{\widehat{r}_{ij\|k}-\mu}{\sigma^2+\sigma_\varepsilon^2})$	not lending
2	$U_{ij}(0) > 0$, $U_{ij}(1) > 0$	$1-\Phi(\frac{\widehat{r}_{ij\|-k}-\mu}{\sigma^2+\sigma_\varepsilon^2})$	lending
3	$U_{ij}(0) < 0$, $U_{ij}(1) > 0$	$\Phi(\frac{\widehat{r}_{ij\|-k}-\mu}{\sigma^2+\sigma_\varepsilon^2}) - \Phi(\frac{\widehat{r}_{ij\|k}-\mu}{\sigma^2+\sigma_\varepsilon^2})$	following
4	$U_{ij}(0) > 0$ and $U_{ij}(1) < 0$	0	anti-following

Table 1: Strategies for bank i lending to bank j

allows us to provide a complete characterisation of the possible equilibrium lending structures and gain some generalizable insights into interbank markets. The number of possible network structures is $2^{N(N-1)}$ and thus for $N = 3$ consists of 64 potential equilibria to consider while for $N = 4$ this becomes an untractable 4096 potential equilibria.

4.1 Individual lending strategies

As we can see from equation (11), the expected returns and risks of bank i lending to bank j depend on the behavior of the remaining bank, k. Banks will lend if the expected utility from doing so exceeds the expected utility from not lending, with the former given by equation (1) and the latter easily being verified to be zero by inserting $a_{ij} = 0$. We can now investigate the different potential outcomes in the lending decision of bank i towards bank j. If $U_{ij}(0) < 0$ and $U_{ij}(1) < 0$, then the bank will not lend as regardless of the behavior of the other bank as the expected utility from doing so is negative. Similarly, if $U_{ij}(0) > 0$ and $U_{ij}(1) > 0$, then the bank will lend as regardless of the behavior of the other bank as the expected utility from doing so is positive. If $U_{ij}(0) < 0$ and $U_{ij}(1) > 0$, then the bank will only lend if the other bank also lends as only with the information that the other bank lends, the expected utility becomes positive. We will refer to this situation as "bank i following bank k". Finally, if $U_{ij}(0) > 0$ and $U_{ij}(1) < 0$, then the bank will only lend if the other bank does not do so as the expected utility from doing so is negative. We will refer to this situation as "bank i anti-following bank k". Table 1 summarizes these situations. We can now determine the probabilities for each of these situations by firstly defining

$$\begin{aligned} \widehat{r}_{ij|k} &\in \{\widehat{r}_{ij}|U_{ij}(1)=0\}, \\ \widehat{r}_{ij|-k} &\in \{\widehat{r}_{ij}|U_{ij}(0)=0\}. \end{aligned} \tag{12}$$

We can solve for $\widehat{r}_{ij|k}$ and $\widehat{r}_{ij|-k}$ numerically. As we can show that $U_{ij}(0)$ is monotonic in $\widehat{r}_{ij}$, the solution for $\widehat{r}_{ij|-k}$ will be unique. While $U_{ij}(1)$ is not monotonic in $\widehat{r}_{ij}$ in general, it is so in the range of realistic parameters, such that the solution will be unique in the relevant range. We furthermore can easily show that $\widehat{r}_{ij|k} \leq \widehat{r}_{ij|-k}$.

Given the definition of $\widehat{r}_{ij|k}$ and $\widehat{r}_{ij|-k}$ in equation (12) it is obvious that $U_{ij}(0) < 0$ is equivalent to $\widehat{r}_{ij} < \widehat{r}_{ij|-k}$ and $U_{ij}(1) < 0$ corresponds to $\widehat{r}_{ij} < \widehat{r}_{ij|k}$. The probabilities for each of the four lending strategies can now easily be determined where the "not lending" corresponds to $Prob(\widehat{r}_{ij} < \widehat{r}_{ij|k} \leq \widehat{r}_{ij|-k})$ and "lending" has a probability of $Prob(\widehat{r}_{ij} > \widehat{r}_{ij|-k} \geq \widehat{r}_{ij|k})$. The strategy "following" has a probability of $Prob(\widehat{r}_{ij|k} < \widehat{r}_{ij} \leq \widehat{r}_{ij|-k})$ and the strategy "anti-following" corresponds to $Prob(\widehat{r}_{ij|k} > \widehat{r}_{ij} \geq \widehat{r}_{ij|-k})$, which is impossible as $\widehat{r}_{ij|k} \leq \widehat{r}_{ij|-k}$ and hence we can neglect this strategy. The probabilities are shown in table 1 and the derivation is straightforward when using the distribution of the returns from equations (2) and (3).

In order to obtain the equilibrium lending structures we will also need to consider the distribution of lending decisions to bank j by banks i and k. We can easily derive that

$$\begin{bmatrix} \widehat{r}_{ij} \\ \widehat{r}_{kj} \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_j \\ \mu_j \end{bmatrix}, \begin{bmatrix} \sigma^2+\sigma_\varepsilon^2 & \sigma^2 \\ \sigma^2 & \sigma^2+\sigma_\varepsilon^2 \end{bmatrix} \right). \tag{13}$$

Table 2 shows the possible strategy combinations and the outcome of the lending decision which we would observe. The corresponding probabilities could easily be derived from the joint distribution in equation (13). With these results we can now continue to apply an algorithm to find all possible equilibria in the coming section.

Strategy (i, j)	Observed outcome (i, j)
(not lending, not lending)	(not lending, not lending)
(following, not lending)	(not lending, not lending)
(lending, not lending)	(lending, not lending)
(not lending, following)	(not lending, not lending)
(following, following)	(lending, lending), (not lending, not lending)
(lending, following)	(lending, lending)
(not lending, lending)	(not lending, lending)
(following, lending)	(lending, lending)
(lending, lending)	(lending, lending)

Table 2: Strategy combinations for banks i and $k$lending to bank j

4.2 Algorithm to determine equilibria

Each bank has three possible strategies, "lending", "not lending", and "following". In a network consisting of three banks there are six possible lending decisions, hence a total of $3^6 = 729$ scenarios have to be considered. For each scenario we can now determine the probability of its occurrence. Using the possible strategy combinations from table 2 we can determine the probability for each pair of banks by applying the joint distribution from equation (13). As r_i is by assumption independently distributed across banks, the probability of a scenario is given by the product of the probabilities for each of the three pairs.

In a scenario those strategies that are "lending" or "not lending" are taken as examined because they do not depend on the action of other banks. Strategies that are "following" are classified as unexamined.

For each scenario we now need to determine the equilibrium outcome by applying the following steps:

1. All examined strategies are fixed at that value and if there are no unexamined strategies we have established a unique equilibrium for that scenario. If there are unexamined strategies, we continue with the next step.

2. If there is an examined strategy from bank k to bank j and an unexamined

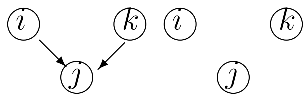
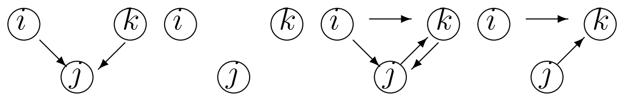
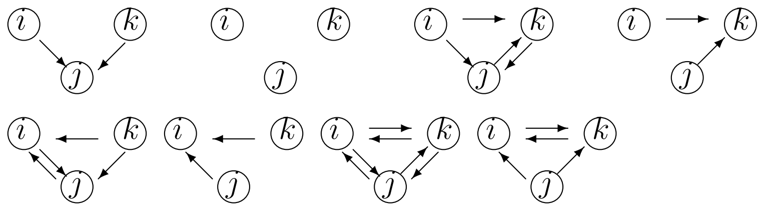
Unexamined strategies	Equilibrium networks
1 unexamined strategy (i and k towards j)	
2 unexamined strategies (i and k for j, i and j for k)	
3 unexamined strategies (i and k for j, i and j for k, j and k for i)	

Table 3: Multiple equilibria with unexamined strategies

relationship from bank i to bank j, then this unexamined relationship is set to the same value as the examined relationship and marked itself as examined and we have established a unique equilibrium for this scenario. If there are any more unexamined strategies, we continue with the final step.

3. If banks i and k both follow a strategy of "following" for lending to bank j, then multiple equilibria will occur, namely both "following" or both "not following". Depending on how many pairs of banks have unexamined strategies, the type and number of equilibria are different and shown in table 3. We assume that each of the possible equilibria has an equal probability of occurrence.

Having now determined the equilibria for each scenario and calculated their probabilities, we now have to aggregate those probabilities for networks that are observationally identical. We thus have established a probability distribution for the equilibria and we can instantly see that each of the 64 possible networks can be an equilibrium network, the probability of its occurrence will vary though with the parameters employed, namely the expected returns μ_i, variance σ^2, signal precision σ_ε and the risk aversion of the banks λ_i.

The following section will now evaluate the properties of the resulting equilibria, firstly for a homogenous banking system and then for a banking system with banks of different expected returns.

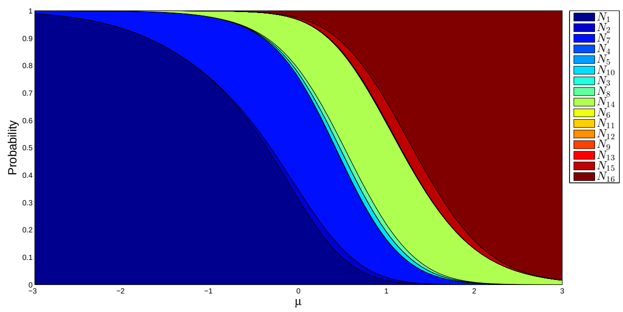
5 Equilibrium structures

Using the procedure outlined above we can now continue to assess the interbank lending that would emerge in equilibrium. We will assess how key variables affect the equilibrium interbank lending behavior of banks firstly in the case of all banks having the same expected returns and differing only in the private signals they receive about the expected returns for lending to other banks. We will then extend this restrictive case to incorporate banks with different returns as a more realistic alternative.

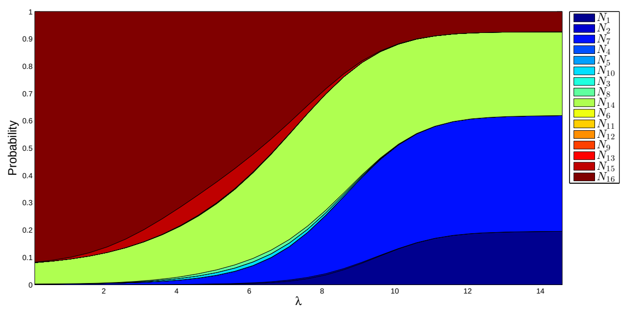
5.1 Homogeneous banks

Let us firstly assume that the expected return of providing interbank loans to banks is identical for all banks, i. e. $\forall i : \mu_i = \mu$. As the private signal banks receive cannot be observed by other banks or an outside spectator, all banks are ex-ante identical and the number of potential networks reduces from 64 to 16 as we can ignore the identity of banks and aggregate networks that are therefore looking alike, i. e. topologically equivalent.

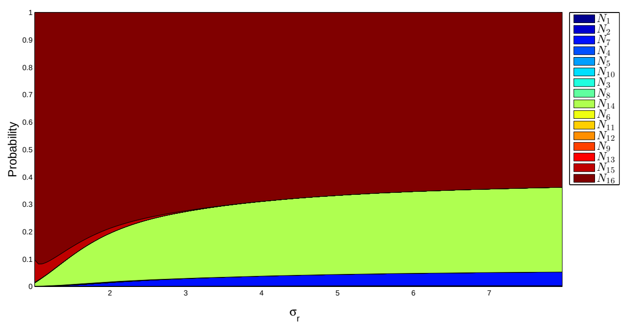
With the aforementioned, each of these 16 interbank lending networks will be an equilibrium for any parameter constellation, the probability of observing a specific network will, however, vary. These probabilities of observing a specific network structure are what we focus our subsequent analysis on. We set $\sigma = 1$ as a normalization of the amount of risk in the banking system, having verified that the results presented here are not substantially affected by this normalization. The parameters we are varying in the following analysis are the risk aversion of banks, λ_i, which for simplicity we assume to be identical across banks, i. e. $\forall i : \lambda_i = \lambda$, the expected returns μ, and the precision of the private signal σ_ε.



(a) Varying μ



(b) Varying λ



(c) Varying σ_r

Figure 1: Probability of equilibrium network structures for varying parameters (base case $\mu = 2$, $\lambda = 2.5$, $\sigma_r = 1.6$)

Holding λ and σ_ε constant, we firstly analyse the impact the expected return μ has on the main equilibrium networks that emerge. Figure 1 shows the probabilities of all 16 networks for a range of expected returns. it is obvious from the figure that the networks that can actually be observed will be dominated by only four of the 16 possible networks, which are shown separately in figure 2. Firstly we observe that for

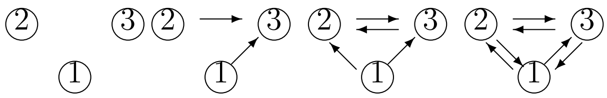


Figure 2: Dominant equilibrium networks

low expected returns the empty network dominates all other equilibrium networks. The low expected returns make the provision of interbank loans unprofitable unless a very large positive private signal is received. Such a large private signal is very unlikely to be received and thus anything but a non lending decision has a small probability. Once the expected return increases, the private signal compensating a large negative expected return becomes more likely. Furthermore the likelihood of a following strategy increases as the private signal itself might not be sufficient to induce lending, but in combination with the lending decision of the other bank this might be sufficient. Hence we will slowly observe the emergence of lending to at least one bank. As the expected return increases even more, it becomes more and more likely that the same will become true for the private signals regarding two banks and we observe the emergence of a third equilibrium in which two banks are lent money. A further increase of the expected return will then make the lending to all three banks more and more likely, first arising from following strategies and then once the expected return is sufficiently positive also based directly on the expected return. We note that the full network emerges only once the expected return is significantly above zero due to the risk aversion of the banks. Once the expected returns are sufficiently positive the entire reasoning reverses.

From this reasoning of the observed dominant equilibrium networks we see that they are those networks that allow a following strategy as shown in table 3. The origin of this result arises from the fact that a following strategy is the most likely observation for expected returns that are neither too large or too small.

For positive expected returns, the likelihood of observing a full network reduces as the risk aversion λ increases as we can see from figure 1. The reasoning is obvious: as the risks are becoming more and more important, it does allow the private signal to be less and less negative in order to generate positive expected utility. For the same reason the empty network becomes more and more likely. Once again we observe an intermediate range with other network structures that allow for following strategies with the same arguments as above. The main difference, however, is that as the risk aversion increases, the empty network does not become dominant but rather the four main network stabilise in fixed proportions. The origin of this observation is that while a higher risk aversion reduces the expected utility of lending, the very same consideration will also be true of the other banks, hence observing another bank lending implies a very high private signal, making the adjustment to the expected return and variance in lemma 3.1 more pronounced, offsetting each other in the variance and expected return and thereby causing the stability of the probabilities of equilibrium network structures as the risk aversion increases.

Looking at the impact of the total risk, σ_r, on the probability of observing specific equilibrium networks, we observe a similar pattern as in the case of increasing risk aversion. The reason here is firstly along the same lines as with risk aversion, but in addition we can also see that the update of expected returns and variance reduces as the variance of the private signal increases due to its much more limited informational content. Hence banks rely less on their private information and observing other banks will also have limited informational value.

In summary, we find that the equilibrium network structures are dominated by four networks that are all consistent with banks adopting a following strategy.

5.2 Banks with different returns

We now relax the assumption that the expected returns of all banks are identical and instead focus on a situation where $0 = \mu_1 \leq \mu_2 \leq \mu_3 = 2$, with other parameter constellations showing comparable results. As the middle-ranking bank increases its expected returns, μ_2, we see from figure 3 that the empty network becomes less likely and the full network more likely. An increase in μ_2 will, ceteris paribus, make the lending to the middle ranking bank more attractive and thus the absence of any lending will become less likely. Similarly the likelihood of a full network will increase. The other two dominant networks, which are identical to the ones identified in the homogeneous case previously, and the argument on their emergence are unchanged.

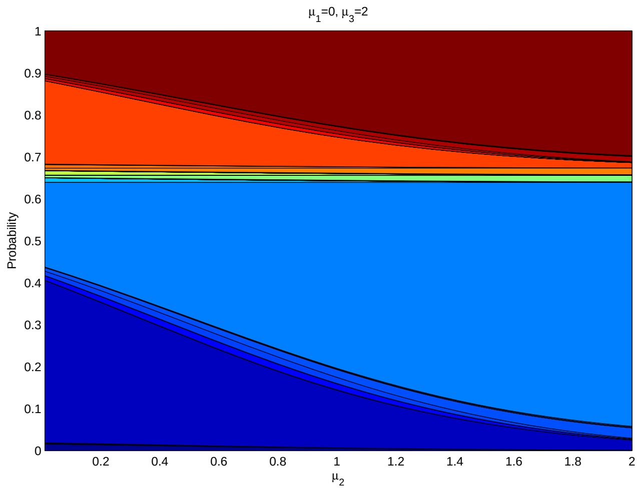
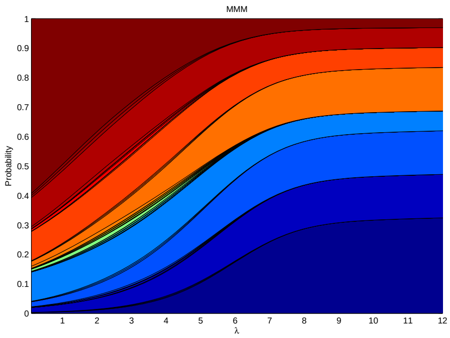


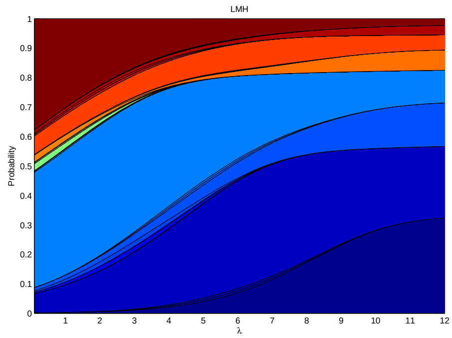
Figure 3: Probability of equilibrium network structures for varying μ_2 ($\lambda = 2$, $\sigma_r = 1.6$)

Assessing the impact of changing the risk aversion of banks is shown in figure 4. Here we observe the same properties as in the homogenous case and the results are only varying to the extent that the incomplete non-empty networks are found. If the mid-ranking bank has a high expected return, we observe that it receiving interbank loans is higher and thus the probability of these network structures is increased accordingly. The same observation we also make when analysing the effect a change of the risk has on the observed network structures as in figure 5.

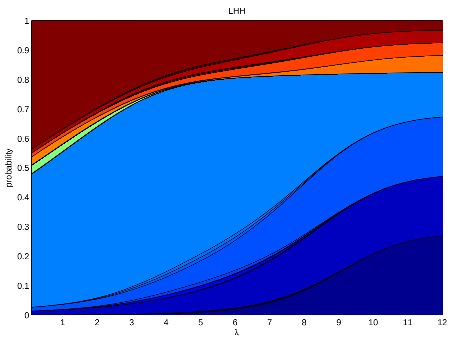
We can thus conclude that the introduction of heterogeneity in the banks' expected returns does not affect the outcome substantially and the results obtained for the homogeneous case can be shown to be robust. We will thus focus on the homogeneous case in the following discussion of the policy implications.



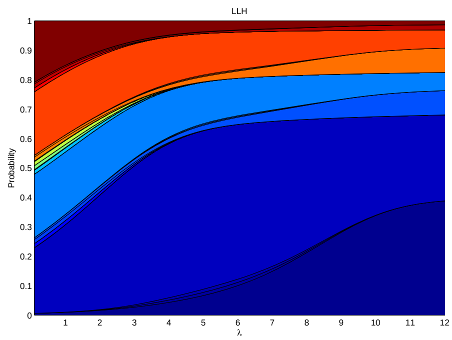
(a) Homogeneous case $\mu_i = 1$



(b) Heterogeneous case $\mu_2 = 1$



(c) Heterogeneous case $\mu_2 = 2$

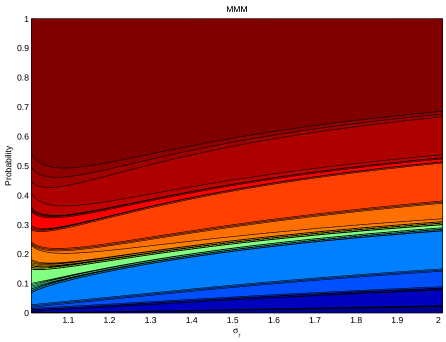


(d) Heterogeneous case $\mu_2 = 0$

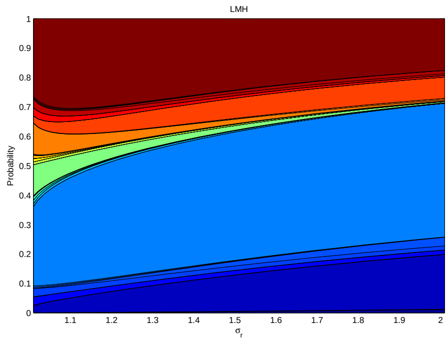
Figure 4: Probability of equilibrium network structures for varying λ ($\sigma_r = 1.6$)

6 Policy implications

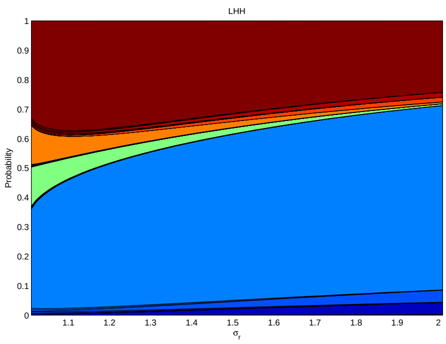
The credit crisis 2007-08 was characterized, among other things, by the withdrawal of interbank lending facilities of banks. Analysis showed that the uncertainty surrounding the solvency of other banks made banks very cautious in advancing new interbank loans or extending the maturity of existing arrangements. We can use our model to explain these observations. If the risk of banks increases our model suggests that the likelihood of networks that show less interbank lending or even its absence become more likely, explaining the reduction in interbank lending that was observed. This effect might have been well exacerbated by an increase in the risk aversion of banks in times their own solvency was questioned as confirmed by our model. Hence even without a reduction in the expected returns, due to the questionable solvency of many banks, we should observe a reduction in interbank



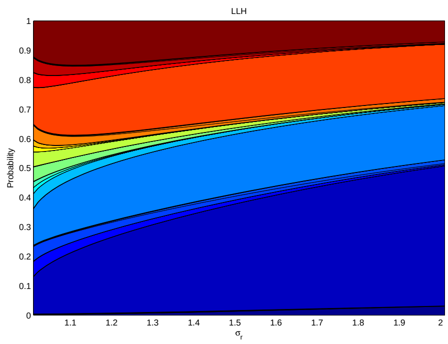
(a) Homogeneous case $\mu_i = 1$



(b) Heterogeneous case $\mu_2 = 1$



(c) Heterogeneous case $\mu_2 = 2$



(d) Heterogeneous case $\mu_2 = 0$

Figure 5: Probability of equilibrium network structures for varying σ_r ($\lambda = 2$)

lending.

From our model we can also deduct the importance of the following strategy in the emergence or absence of interbank lending. Hence the existence of interbank lending will to a large extend depend on expectation formation, and thus it is important to maintain trust in the solvency of banks. A reduction in the quality of the private signals banks have about each other will also be detrimental to the existence of a flourishing interbank market. Any regulator might want for this reason seek to ensure that information on a banks' solvency is easily available as to reduce the risks banks expose themselves to.

7 Conclusions

We provided a model of interbank lending where banks seek to maximize expected utility in the presence of uncertainty regarding the risks of a counterparty bank. Banks assess the risk of other banks by relying on a public signal, their individual private signal as well as extracting information from the lending behavior of other banks. We showed that in equilibrium four network structures of interbank lending dominate, the empty network (no lending), the full network (all banks lend to each other), a network in which one bank receives interbank loans from the other banks, and a network where two banks receive interbank loans from the other banks. These network structures were arising mainly from a "following" strategy in which a bank would only lend if the other bank would also do so, providing equilibria that are vulnerable to small shocks that can change the equilibrium structure easily.

The analysis of the model presented here was limited to banking systems with only three banks. An extension to include more banks is in principle straightforward but comes at the cost of significantly increased computational complexity such that additional constraints on the network structure would have to be imposed to make it tractable. A further extension might be that the expected returns are made endogenous to the network structure and the exposure of the bank to interbank loans, thus depending on the network structure itself. Using such extensions would allow us to provide a more general equilibrium of the interbank lending network.

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Chapter 3

Essay 2 - Balancing Liquidity and Profitability through Interbank Markets: Endogenous Interest Rates and Network Structures

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Contents

Balancing Liquidity and Profitability through Interbank Markets: Endogenous Interest Rates and Network Structures

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Abstract

This paper develops a model of interbank lending based on liquidity and profitability considerations of homogeneous banks. We derive the reservation prices of interbank lending and its properties before exploring how, due to an idiosyncratic liquidity shock, banks engage in bilateral lending to form an interbank network. We establish that the resulting networks exhibit realistic properties, including a core-periphery structure. Banks in the core and the periphery of this network do not only differ in the amounts of interbank lending and borrowing, but also in the interest rates applied to their transactions.

Keywords: *Interbank lending, interbank network, core-periphery structure, interbank interest rates, bilateral transactions*

1 Introduction

Interbank networks have been shown to be important transmitters of financial distress and the precise structure of these networks determines the extent of any such contagion. Given the relevance of the network structure it is important to understand how these are formed as only then can regulators assess the impact of any regulatory measures that would lead to a network structure being more robust to any banks failing. Currently the literature on how interbank networks are formed

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is quite limited, mostly relying on ad hoc assumptions that generate heterogeneity among banks which then manifests itself in a core-periphery structure.

In this paper we seek to address the formation of such interbank networks by developing a theoretical model that replicates key properties observed empirically. Our main contribution here is the introduction of a framework in which banks balance their demands from liquidity with their desire for high profitability when reacting to an idiosyncratic liquidity shock by engaging in bilateral transactions. The interbank lending rates in our model are determined endogenously based on the preferences of the individual banks. We are also able to investigate differences between banks based on their position in the interbank network that emerge endogenously.

The absence of credit risk in our model allows us to consider the emergence of interbank networks prior to the financial crisis 2007-2008 when credit risk between banks was largely seen as irrelevant. With liquidity considerations being seen as more important in the aftermath of the crisis, our model also remains relevant in the post-crisis period. It allows us to focus on the consequences of the management of liquidity through interbank markets, which is the primary reason for their existence as introduced in Allen & Gale (2000).

We proceed with a review of properties of interbank networks and the existing models of their formation in the coming section. Section 3 then introduces the model and derives some key variables and properties of the resulting interest rates with all proofs presented in the appendix. How we use computer experiments to assess the interbank network structure is outlined in section 4 before section 5 focuses on the evaluation of the key properties of these resulting interbank networks. The determinants of a bank's position in the network are discussed in section 6 and section 7 concludes our findings.

2 Literature Review

A number of studies have established some key properties of interbank lending networks, see e.g. Boss et al. (2004), Inaoka et al. (2004), Soramäki et al. (2007), Iori et al. (2008), Bech & Atalay (2010), amongst others. Firstly they find the interbank network to be sparse, i. e. only a small fraction of the possible lending relationships are actually observed. Secondly, the degree distribution (the number of lending and borrowing relationships a bank has) exhibits a fat tail, where evidence of a power law decay is reported in some studies. The third property commonly found is that interbank lending networks exhibit "small-world" properties, hence the distance between any two banks is small. Finally, interbank lending networks are showing dissassociativity (negative associativity), meaning that highly connected banks tend to be connected to less highly connected banks and vice versa.

Starting with Craig & Von Peter (2014) and confirmed in Langfield et al. (2014) and Fricke & Lux (2015), it has been established that interbank lending markets exhibit a core-periphery structure. Here a small number of highly interconnected banks form the core and the other banks only borrow to and lend from these core banks, but rarely does borrowing or lending happen between periphery banks themselves. These studies find such a structure for a variety of markets, including Germany, the UK, the Netherlands, and the Italian e-Mid market; they establish that the core consists of a small fraction of banks in the network.

Works by Nier et al. (2007), Gai et al. (2011), Lenzu & Tedeschi (2012), Krause & Giansante (2012), and Teteryatnikova (2014), amongst others, show the relevance of the network structure for the spread of bank failures and hence systemic risk. Given this relevance of the network structure, it is of paramount importance to gain an understanding how banks form these and what its determinants are.

While quite a few models exist to generate networks with properties as outlined above, i. e. sparse networks with a fat tailed degree distribution, disassociativity, and small-world properties, this is much less commonly achieved for a core-periphery structure. Such models are highly unsatisfactory as they are based on statistical models rather than the actual behaviour of individuals - banks in our case - and thus do not provide a framework to assess the impact of incentives and regulation on the properties of interbank networks. Contributions by Farboodi (2014) and in't Veld et al. (2014) employed heterogeneous banks to achieve a core-periphery structure. Specifically, Farboodi (2014) showed that banks with risky investment opportunities will form the core, while those lacking such opportunities populate the periphery. in't Veld et al. (2014) require banks of different sizes to establish such a network structure. Lux (2015), on the other hand, uses a model including reinforcement learning to build trust among banks that leads to a core-periphery structure based on assessed credit risk.

Similarly, Hałaj & Kok (2015) use a framework considering credit risk, but do not report the statistical properties of the resulting interbank networks in detail. Also based on credit risk arising from lending to non-bank counterparties is the contribution by Cohen-Cole et al. (2010), while Castiglionesi & Navarro (2016) show the emergence of a core-periphery network in the presence of such counterparty risk. A similar framework to our model, but in OTC asset markets, is developed in Wang (2016) where also a core-periphery structure emerges and the core is formed of endogenously emerging dealers.

Our paper contributes to this literature on the formation of realistic interbank lending networks, including a core-periphery structure. We do so without assuming heterogeneous banks and instead rely on the bilateral interactions of banks seeking to balance liquidity and profitability. This neglect of credit risk allows us to understand how the interbank network establishes itself from the very purpose of its existence, the managing of liquidity as introduced in Allen & Gale (2000).

In order to initiate trades in the interbank market, we introduce idiosyncratic shocks to banks' liquidity buffers. One may further explore as to what causes these liquidity shocks. Admittedly, heterogeneity in bank size or investment opportunities can potentially influence the distribution of cash between banks as well. However, they are not the only reasons why banks differ in liquidity levels. Other possible causes also play important roles such as imbalances arises from funding withdraws, or the use of credit lines. Thus, the assumption of random liquidity shocks is not equivalent to that of heterogenous sizes, or differing investment opportunities. Moreover, in our model the motive to trade differs from previous models as we highlight banks' needs for both liquidity and profitability. Core banks are established through bilateral trading with observable rates that have not featured in previous models.

3 A model of interbank lending and borrowing

We seek to analyze the unsecured short-term interbank market, i. e. the overnight market for funds between banks. This allows us, as an approximation, to neglect the default risk associated with such interbank loans and focus on the effect of liquidity management and profitability instead. As a consequence of considering short-term lending and borrowing only, we can take all other parts of a bank as exogenously given, such as the interest rates applied to loans and deposits and their sizes.

We model the preferences of banks for cash reserves and profitability, and based on this derive the reservation prices of borrowing and lending. These prices are then used to initiate bilateral lending agreements between banks looking to improve their overall utility level. This section will introduce the actual model used in our analysis of the resulting interbank market structure in the coming section.

3.1 Behaviour of individual banks

We assume a stylized balance sheet of banks $i = 1, \ldots, N$, consisting of cash reserves $\mathbf{R}_i$, interbank loans $\mathbf{L}_i$, and external loans to non-financial debtors $\mathbf{C}_i$ on the asset side. The liabilities consist of external deposits by non-financial creditors $\mathbf{D}_i$, interbank deposits $\mathbf{B}_i$, and equity $\mathbf{E}_i$. The total size of the assets is denoted by $\mathbf{A}_i$.

Bank i			
Cash reserves	$\mathbf{R}_i$	Deposits	$\mathbf{D}_i$
Interbank lending	$\mathbf{L}_i$	Interbank borrowing	$\mathbf{B}_i$
Loans	$\mathbf{C}_i$	Equity	$\mathbf{E}_i$
Total assets	$\mathbf{A}_i$		$\mathbf{A}_i$

Figure 1: Bank i's stylized balance sheet

For reference this stylized balance sheet is as shown in figure 1. The inclusion of other assets and liabilities would not alter the results of our analysis here.

We can now define the liquidity ratio ρ_i as well as the leverage ratio λ_i as

$$\rho_i = \frac{\mathbf{R}_i}{\mathbf{D}_i + \mathbf{B}_i}, \tag{1}$$

$$\lambda_i = \frac{\mathbf{A}_i}{\mathbf{E}_i}. \tag{2}$$

Banks pay and receive interest on their balance sheet items at different rates, depending on the position. r^f is the risk-free interest rate at which cash reserves are remunerated, r_i^C is the average rate on external loans granted and r_i^D is the average rate on external deposits, where $r_i^D < r^f < r_i^C$. The average rates on interbank lending and borrowing are denoted by r_i^L and r_i^B, respectively. These rates are determined as

$$r_i^L = \sum_{j=1}^{N} \frac{\mathbf{L}_{ij}}{\mathbf{L}_i} r_{ij}^L, \tag{3}$$

$$r_i^B = \sum_{j=1}^{N} \frac{\mathbf{B}_{ij}}{\mathbf{B}_i} r_{ij}^B, \tag{4}$$

where r_{ij}^L (r_{ij}^B) denotes the interest rate of bank i lending (borrowing) the amount L_{ij} (B_{ij}) to (from) bank j. We also define $\mathbf{L}_i = \sum_{j \neq i} \mathbf{L}_{ij}$ and $\mathbf{B}_i = \sum_{j \neq i} \mathbf{B}_{ij}$. We do not allow banks to lend to themselves such that $\mathbf{L}_{ii} = \mathbf{B}_{ii} = 0$ and obviously if bank i lends to bank j it is that bank j borrows from bank i, hence $\mathbf{L}_{ij} = \mathbf{B}_{ji}$ and $r_{ij}^L = r_{ji}^B$.

We measure a bank's profitability by its return on equity r_i^E, which, assuming no revenue and costs besides the interest payments, is given by

$$r_i^E = \frac{\mathbf{R}_i r^f + \mathbf{C}_i r_i^C + \mathbf{L}_i r_i^L - \mathbf{D}_i r_i^D - \mathbf{B}_i r_i^B}{\mathbf{E}_i}, \tag{5}$$

where additional revenues and costs can be included without affecting the outcomes reported here.

We assume that banks prefer larger cash reserves and higher profitability. The preference for higher cash reserves can be interpreted as a protection against sudden withdrawals of depositors or interbank lending and hence we use the liquidity ratio ρ_i as a measure for the size of these cash reserves. The profitability of a bank is measured by its return on equity r_i^E and naturally banks seek to maximize these returns. These two objectives of the bank need to be balanced in the utility function, which we assume to follow a Cobb-Douglas form:

$$U_i\left(\rho_i, 1 + r_i^E\right) = \gamma_i \rho_i^{\theta_i}\left(1 + r_i^E\right)^{1-\theta_i}, \tag{6}$$

where $\gamma_i > 0$ and $0 \leq \theta_i \leq 1$ denotes the strength of the preferences for liquidity relative to profitability. Banks can adjust their cash reserves in the short run only via interbank lending and borrowing as we assume that all other balance sheet positions are fixed.

Our approach here is similar to that of Stoll (1978) in that banks seek to optimize their inventory holdings (cash reserve) and balance this against the profits made from trading (return on equity). While in the original work this balance was maintained due to the risky assets that were traded, we instead recognize the preferences of banks for liquidity directly. However, we employ similar ideas by determining the reservation prices (interest rates) at which banks are willing to borrow and lend funds for a given amount and using this as the basis for a bilateral agreement. In this context we also show how the size of the loan affects interest rates and the differences in the reservation prices between borrowing and lending.

The motivation for the use of a utility function where banks prefer both liquidity and profitability can be explained as follows. It is quite easy to understand why banks favour higher profits, yet not so to see why they seek more cash. The main reason is that banks would like to hold more cash in response to unexpected cash outflows and potential funding risk. Capital is a safety buffer since it covers losses and allows banks to recover from them. However, the recent financial crisis has shown that liquidity problems can also contribute to a bank's distress. A bank can still fail, even when holding assets more than sufficient to cover its liabilities, because its assets are illiquid and its liabilities have short-term maturities. To make matters worse, depositors and other funders may lose confidence in the bank. In this scenario, funding risk rather than default risk is most detrimental to the bank. Thus, we highlight the importance of the "liquidity" buffer in our model.

It is worth pointing out that the need for liquidity buffer as a protection is strongest when banks do not hold much cash. Thus the utility function used is relatively weak to describe a banking system where banks are holding plentiful cash and do not desire any more cash. However, what we intend to study in this essay is a well functioning banking system during "normal" times. In such a system, it is common that banks do not hold much cash since they have abundant investment opportunities. Therefore, we believe the utility function still fits the aim of our study. Moreover, this disadvantage can also be partially addressed when the use of our utility function is combined with interbank trading. The main reason why banks are reluctant to hold extra cash is that there are opportunity costs for doing so. Otherwise, banks may be indifferent about having more cash or not. The interbank trading implies acquiring more cash is costly for banks. More specifically, when a bank increases its liquidity, it has to borrow some cash from other banks at a rate higher than risk free rate (as explained in Lemma 2), which means the bank will see a decrease in its return on equity as a cost of holding more cash. Note that a decrease in return on equity on its own reduces the bank's utility. Therefore, a clear trade-off between liquidity and return makes sure the bank substitutes one for another only when its utility can be improved. In our model, banks with a lot of cash actually do not increase their cash level further because at the available borrowing rate the effect of the decrease in return outweighs that of the increase in liquidity buffer.

Proposition 1 (Existence of optimal cash reserves)**.** *If there is no arbitrage opportunity in the interbank lending market, then a unique optimal reserve ratio exists. Banks with reserve ratios below (above) this level would only seek to increase interbank borrowing (lending), but not both interbank lending and borrowing. Banks holding the optimal reserve ratio would not seek any interbank lending or borrowing unless their profitability increases.*

This proposition establishes that a unique optimal reserve ratio exists and that banks do not adjust their reserves by increasing interbank lending and interbank borrowing simultaneously, even if by different amounts. The reasoning here is that while such a strategy would increase the utility by obtaining the optimal reserve ratio, the no arbitrage condition implies higher costs of borrowing, compared to lending, thus reducing the profitability of such a strategy. Another consequence of this proposition is that we can ensure that banks do not seek to increase their total assets unnecessarily without the need for capital regulation.

In the special case of having no interbank lending or borrowing we can determine the optimal reserve ratio analytically and derive its properties as shown in the following lemma:

Lemma 1 (Optimal cash reserves without interbank lending)**.** *If* $\mathbf{L}_i = \mathbf{B}_i = 0$ *then* $\rho_i^* = \frac{\theta_i}{r_i^C - r^f}\left(r_i^C - r_i^D + \frac{\mathbf{E}_i}{\mathbf{D}_i}\left(1 + r_i^C\right)\right)$. *For a given* $\mathbf{A}_i$, *we find that* $\frac{\partial \rho_i^*}{\partial \lambda_i} < 0$, $\frac{\partial \rho_i^*}{\partial r_i^D} < 0$, $\frac{\partial \rho_i^*}{\partial r_i^C} < 0$, *and* $\frac{\partial \rho_i^*}{\partial r^f} > 0$.

Banks with a higher leverage will hold less cash reserves. This result is due to the fact that a higher leverage implies higher profits from the differences between borrowing and lending to external non-banks, shifting the emphasis towards higher profitability and thus reducing cash reserves. We furthermore observe that the optimal cash reserves are reducing in the lending rate due to the better investment opportunities this implies; an increased deposit rate increases the costs of funding to banks and thus incentives banks to hold less cash in order to maintain their profitability. The higher return on cash reserves through higher risk-free rates increases their holdings.

While the above proposition shows that banks would seek to move towards their optimal cash holding, whether such a move is actually utility enhancing will depend on the interest rates involved. We can now determine the interest rates at which a bank is indifferent between no lending/borrowing conducting an interbank borrowing (r_i^a) or lending (r_i^b) of size Q. The following proposition shows these reservation prices.

Proposition 2 (Reservation prices)**.** *If* $Q < \mathbf{R}_i < \mathbf{D}_i + \mathbf{B}_i$ *and* $r_i^E > -1$*, the reservation prices of bank i* (r_i^a, r_i^b) *are given by*

$$r_i^a = r^f + \left(1 - \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q}\left(1 + \frac{Q}{\mathbf{D}_i + \mathbf{B}_i}\right)\right)^{\frac{\theta_i}{1-\theta_i}}\right)\frac{\mathbf{E}_i\left(1 + r_i^E\right)}{Q} \tag{7}$$

$$r_i^b = r^f + \left(\left(\frac{\mathbf{R}_i}{\mathbf{R}_i - Q}\right)^{\frac{\theta_i}{1-\theta_i}} - 1\right)\frac{\mathbf{E}_i\left(1 + r_i^E\right)}{Q} \tag{8}$$

The following lemma establishes that a bank charges more for lending than it is willing to pay for borrowing and that both rates will exceed the risk free rate. The reason that borrowing rates are above the risk free rate is that the increase in cash reserves increases the utility, hence the borrowing costs must exceed the benefits generated from them, the risk free rate, such that a bank is indifferent between borrowing and not borrowing. Similarly, lending reduces the utility of a bank due to lower cash reserves, thus they must be compensated through a lending rate higher than the benefits of the cash reserves lost, measured by the risk free rate. Due to the utility function used, banks suffer a utility loss from reduced cash reserves larger than the gain from an equally sized increase, hence the lending rate is higher than the borrowing rate. This also prevents any arbitrage opportunities for banks.

Lemma 2 (Relationship of reservation prices)**.** *We find that* $r^f < r_i^a < r_i^b$*.*

Lemma 3 (Partial derivatives of reservation prices)**.** $\frac{\partial r_i^a}{\partial \rho_i} < 0$, $\frac{\partial r_i^b}{\partial \rho_i} < 0$, $\frac{\partial r_i^a}{\partial \lambda_i} < 0$, $\frac{\partial r_i^b}{\partial \lambda_i} < 0$, $\frac{\partial r_i^a}{\partial Q} < 0$, $\frac{\partial r_i^b}{\partial Q} > 0$ $\frac{\partial\left(r_i^b - r_i^a\right)}{\partial \rho_i} < 0$, $\frac{\partial\left(r_i^b - r_i^a\right)}{\partial \lambda_i} < 0$, $\frac{\partial\left(r_i^b - r_i^a\right)}{\partial Q} > 0$

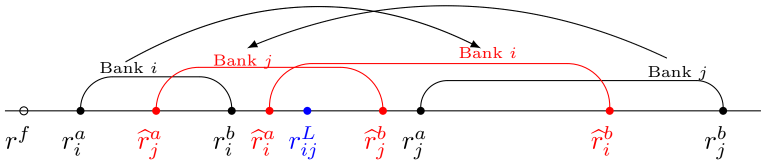


Figure 2: Illustration of change of reservation rates after a transaction.

From lemma 3 we find that larger cash reserves reduces both borrowing and lending rates as banks are more willing to lend and less willing to borrow in this situation, i. e. the acquisition of additional cash becomes less attractive, in accordance with the result on inventory positions in Stoll (1978). Unlike in dealer markets, the spread between borrowing and lending rates reduces as banks are willing to accept lower profits from interbank lending in order to offset their suboptimal cash reserves due to the afore mentioned asymmetry implied by the utility function. A higher leverage allows banks to generate profits from larger quantities of borrowing to non-banks and deposits, reducing the need to increase profitability from interbank transactions and therefore reducing the interest rate (moving it closer to the risk free rate) and the spread. Finally, in line with Stoll (1978) we find that larger interbank loans require lower borrowing rates and higher lending rates, thus increasing the spread, due to the larger impact such loans have on the cash reserves of the bank.

We can also show that if two banks agree on lending, their reservation prices change not so much that another transaction reversing the original lending is possible. This means that banks will not inflate their balance sheets unnecessarily with reciprocal loans.

Lemma 4 (No lending reversal)**.** *If bank i lends to or borrows from bank j, they would not transact with each other in the opposite direction immediately afterwards, ceteris paribus, provided their preferences are identical.*

As illustrated in figure 2, after a transaction, a lender's reservation price for new borrowing is lower than the borrower's reservation rate for new lending, thus making a reversal of the transaction through a new loan unfeasible.

3.2 Interbank trading

As proposition 1 states that banks have no desire to engage in interbank lending and borrowing if they hold the optimal cash reserves, we expose each bank to an idiosyncratic liquidity shock, e.g. arising from non-bank depositors re-allocating their holdings between banks. We ensure that the average cash reserves are the optimal cash reserves as detailed in lemma 1. As cash reserves are restricted to be in the interval $[0;1]$, we chose a Beta distribution such that $\rho_i \sim \text{Beta}\left(\frac{\rho_i^*}{1-\rho_i^*}\beta, \beta\right)$, ensuring the mean as indicated and leaving only a single parameter, $\beta > 0$, to be freely chosen to affect the shape of the distribution.

Facing this idiosyncratic liquidity shock, banks now seek to improve their utility by adjusting their reserve ratio as well as trying to improve their profitability. The trading mechanism is such that no centralized market exists, but borrowing and lending needs to be arranged bilaterally between two banks. We restrict the size of each borrowing and lending to Q and if banks want to borrow or lend larger quantities they can do so in multiple transactions. We assume that prices at which banks are willing to borrow or lend are not public knowledge, but only are revealed if another bank enquires its prices, in line with actual interbank markets. A bank i will ask other banks randomly for the price at which they would be willing to borrow or lend. If this price is below (above) its reservation price for borrowing (lending), the bank will agree on an interbank loan at the price this bank has quoted. The price a bank quotes will be the reservation price; using different pricing rules proved to provide similar results to those reported here, e.g. seeking the average of the two reservation prices, or quoting prices that are below/above the reservation prices.

After a transaction both banks involved update their reservation prices. The banks then continue seeking the quote of an additional bank. A bank remembers any past prices that have been quoted and if in light of the changed reservation prices previous quotes become attainable these are now taken up, such that the best prices are taken up first. This process continues until all banks have been asked for a quote or a maximum of N transactions have been agreed. After N transactions or when all banks have been asked for a quote the bank will restart the process. The process then continues to the next randomly selected bank until all banks have been chosen. Any bank entered the market earlier can at that stage engage in additional transactions in random order following the same rules until no further transactions between banks are possible.

4 Model evaluation

The reservation prices of the above model and their properties can be easily derived using proposition 2. It is, however, not easily possible to derive the properties of the resulting interactions between banks. While we would be able to derive the "average" properties from the resulting network of interbank lending, such as the average degree or connectedness, the specific structure of this network which we are interested in cannot be derived analytically. Specifically we are interested in the emergence of a so called core-periphery structure, which consists of a small number of highly connected banks (the core) and a large number of banks that are only connected to banks in the core (the periphery), as well as any differences between these two types of banks. In order to analyse the network properties, we therefore conducted a computer experiment of the individual interactions between banks as detailed in the model and analysed the resulting individual networks.

It is common in many models that differences between banks emerge as the result of differences between banks, e. g. different preferences, costs, or liquidity needs. With such heterogeneity it is not surprising to observe that banks obtain different positions in the network. Here we are, however, interested in the emergence of any such difference if banks are homogenous and investigate how the interactions between individual banks shape this network structure. To this end we assume that for each simulation run all banks are ex-ante identical by having the same preferences ($\theta_i = \theta$), the same size ($\mathbf{A}_i = \mathbf{A}$), same balance sheet structure ($\lambda_i = \lambda$), interest rates ($r_i^C = r^C$, $r_i^D = r^D$), and no initial interbank loans ($\mathbf{L}_i = \mathbf{B}_i = 0$). The only difference between banks will be the cash reserves ρ_i (and necessary adjustment to the amount external loans to ensure the total assets are matching) by introducing idiosyncratic liquidity shocks as detailed in the previous section. All other parameters are chosen randomly for each run from a distribution as indicated in table 1.

By assuming banks are homogenous in most aspects apart from their initial liquidity buffers, we are not assuming heterogeneity in these aspects are not important for the emergence of interbank network features such as the core-periphery structure. On the contrary, we agree they play key roles in determining whether a bank is core or not. And in real markets, banks are indeed quite heterogenous unlike what we assume in this essay. The advantage of applying such an assumption is that the model gives a different perspective. It allows us to explore the emergence of some interbank network features even without heterogeneity in size or investment opportunities. Also, the model serves as a good starting point and can be easily extended to introduce size heterogeneity based on this framework, and the emergence of a core-periphery structure is likely to be further strengthened as a consequence, given we show that a core-periphery structure emerges without the elements of heterogeneity often assumed in the previous literature, i.e. it materializes endogenously without assumed heterogeneity in key bank properties.

For the identification of the core-periphery structure of a network, we follow Lip (2011). The idea is to partition the network into these two groups such that the core is maximally connected while the periphery has no connections between themselves and only connections to this core. If we have N_c banks in the core $\mathcal{C}$, a perfect core would have $N_c(N_c - 1)$ connections, as we do not allow for banks to lend to themselves. The perfect periphery would have no connections with each other and all connections would only be with the core. A formal definition of core-periphery structures can be found in Borgatti & Everett (2000). We can now define an error score as the difference between this perfect core-periphery structure and the actual

$U(a;b)$ denotes a uniform distribution with a lower limit of a and upper limit of b. Beta denotes the beta distribution with the parameters as indicated.

Parameter	Symbol	Distribution
Assets	$\mathbf{A}$	fixed at 100
Preferences	θ	$U(0;0.1)$
Leverage	λ	$U(2;100)$
Number of banks	N	$U(1;1000)$
Risk free rate	r^f	$U(0.025;0.125)$
Deposit rate	r^D	$U(0;0.025)$
Loan rate	r^C	$U(r^f;0.2)$
Interbank loan size	Q	$U(0;2)$
Distribution of liquidity	β	$U\left(1-\rho^*;9(1-\rho^*)\right)$
Cash reserve ratio	ρ_i	Beta $\left(\frac{\rho^*}{1-\rho^*}\beta,\beta\right)$

Table 1: Parameter selection for simulations

connections, E_{CC} for the errors in the core and E_{PP} in the periphery. Any non-existent connections between the core and periphery are ignored. If we denote a dummy variable c_{ij} that takes a value of 1 if bank i gives an interbank loan to bank j and zero otherwise, we obtain

$$E_{CC}(\mathcal{C}) = N_c(N_c-1) - \sum_{i,j\in\mathcal{C}} c_{ij}, \tag{9}$$

$$E_{PP}(\mathcal{C}) = \sum_{i,j\notin\mathcal{C}} c_{ij}. \tag{10}$$

The total error score is then given by

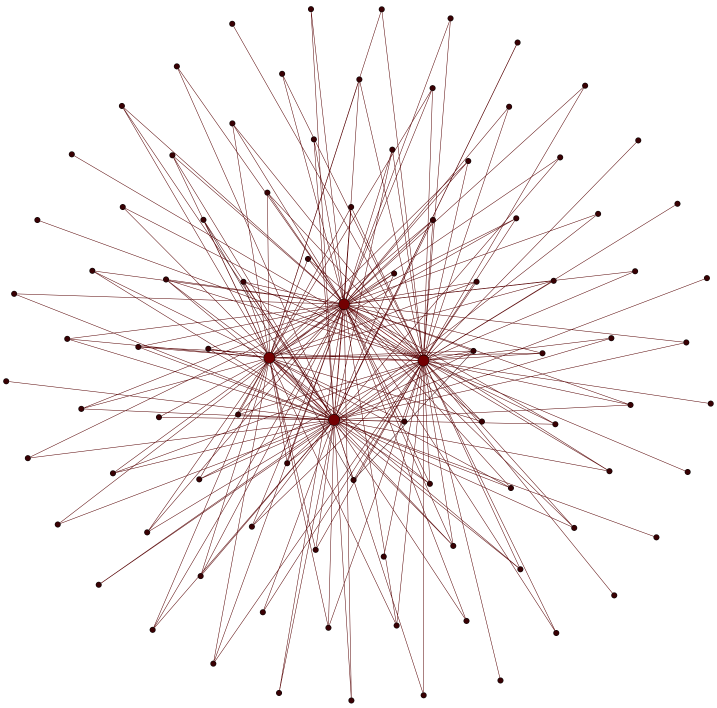
$$E(\mathcal{C}) = \frac{E_{CC}+E_{PP}}{\sum_{i,j} c_{ij}} \tag{11}$$

and the core determined such that

$$\mathcal{C}^* \in \arg\min E(\mathcal{C}). \tag{12}$$

It is common to identify an error score of below 0.5 with a core periphery structure. For interbank networks empirically an error score in the region of 0.3 is commonly found. Other algorithms as developed in Holme (2005) or Da Silva et al. (2008) lead to similar results.

Using a total of 10,000 banking systems, we can now evaluate the properties of the resulting networks in the coming two sections. Some computer experiments will



Parameter constellation: $\theta = 0.0280$, $Q = 1.4424$, $N = 95$, $\frac{1}{\lambda} = 0.0347$, $r^f = 0.0517$, $r^D = 0.0234$, $r^C = 0.0953$, $\beta = 9.0478$. The largest component shown consists of 91 banks and has 4 core banks.

Figure 3: Visualization of an interbank network structure

produce too few interbank loans or the number of banks is too small to for a core-periphery structure to emerge. We will exclude such networks from our analysis and investigate only the largest connected component, i. e. exclude those banks that engage in neither interbank borrowing or lending.

5 Emergence of a core-periphery structure

Conducting computer experiments as outlined in the previous section, we can easily see from figure 3 a core-periphery structure with the four inner banks forming the core. This sample result gives us a clear indication that core-periphery structures emerge in our model from the interactions between banks. A more thorough statistical analysis of this result using all computer experiments is shown in the histograms of figure 4 and the descriptive statistics in tables 2 and 3.

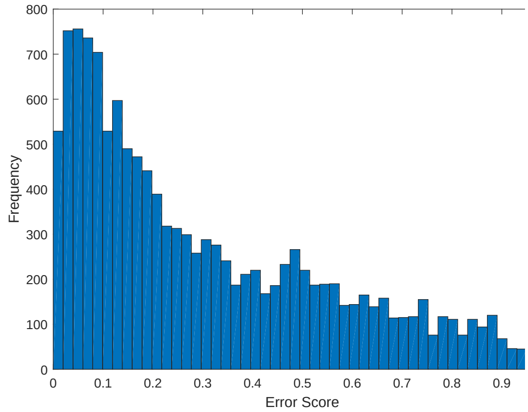
We can see clearly from these descriptive statistics and histograms that the vast majority of interbank networks exhibit a core periphery structure. While not all networks will exhibit realistic properties consistent with actual interbank lending networks, including a core-periphery structure, it is clear that these properties easily emerge, even when banks are homogenous, apart from the size of the idiosyncratic liquidity shock. The core size is low as in all real interbank networks.

We also see that the networks exhibit other realistic properties that are commonly found, namely a low density, i.e. only relatively few banks are lending to each other, a strongly negative assortativity, fat tails of the degree distribution and short average path lengths. The clustering coefficient is relatively low, though, compared to real interbank markets, but we note that for networks with a low error score, i. e. a more pronounced core-periphery structure, this measure actually increases and moves into a more realistic range. Figure 5 shows these relationships and we see that as the error score approaches realistic levels of approximately 0.2-0.3, see Craig & Von Peter (2014) and Fricke & Lux (2015), all other reported network measures also reach levels that are consistent with those reported for actual interbank lending networks.

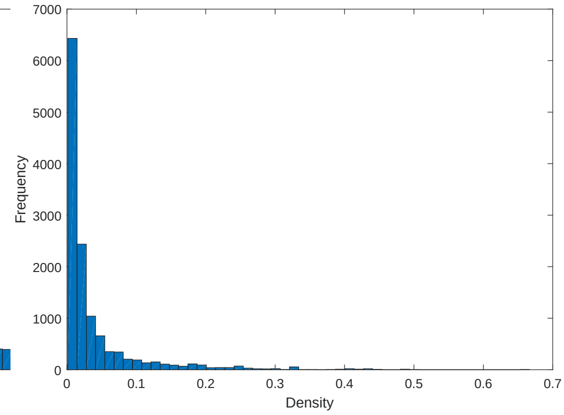
Banks in the core are borrowing at lower rates than those in the periphery and are lending at higher rates. Looking at table 2, we see that periphery banks lend to core banks at a very low interest rate while receiving very unfavorable terms from core

	Mean	Median	Standard deviation	Minimum	Maximum
Size of largest component/Number of banks	0.8109	0.9077	0.2320	0.0038	1.0000
Error score	0.3015	0.2197	0.2517	0.0000	0.9897
Density	0.0380	0.0145	0.0624	0.0013	0.6667
Core Size/Size of largest component	0.0527	0.0219	0.0763	0.0022	0.5000
Lending rate Core to Core	0.2480	0.1729	0.3955	0.0013	12.9345
Lending rate Core to Periphery	0.6203	0.2637	2.8419	0.0014	102.8615
Lending rate Periphery to Core	0.0683	0.0613	0.0601	0.0001	1.7918
Lending rate Periphery to Periphery	0.1164	0.1037	0.1426	0.0007	3.2973
Assortativity In-In	-0.5335	-0.5734	0.2037	-1.0000	0.0648
Assortativity In-Out	-0.4280	-0.4776	0.1730	-1.0000	0.0456
Assortativity Out-In	-0.5130	-0.5647	0.2229	-1.0000	0.2469
Assortativity Out-Out	-0.6084	-0.6395	0.2190	-1.0000	0.0000
Average shortest path	2.2412	2.1718	0.3435	1.0000	4.9409
Average normalized eigenvector centrality of the core	0.0294	0.0195	0.0374	0.0014	0.4142
Global clustering coefficient	0.0409	0.0159	0.0729	0.0000	1.0000
Average Local Directed Clustering Coefficient	0.0771	0.0845	0.0421	0.0000	0.3172
Average Local Undirected Clustering Coefficient	0.4088	0.4589	0.2397	0.0000	1.0000

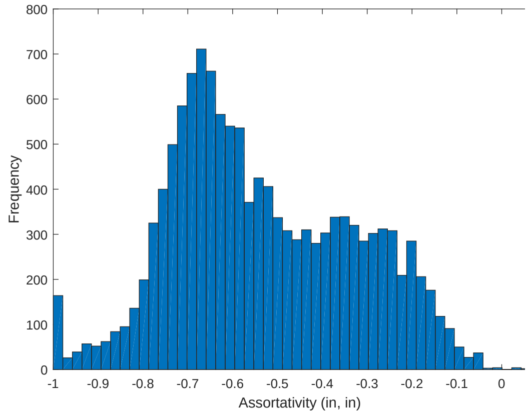
Table 2: Descriptive statistics of network properties of the interbank lending network



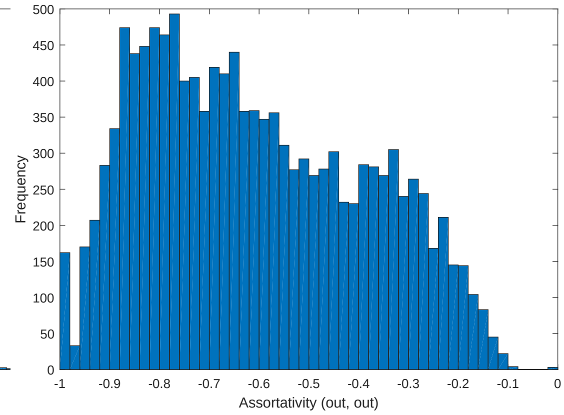
(a) Error score



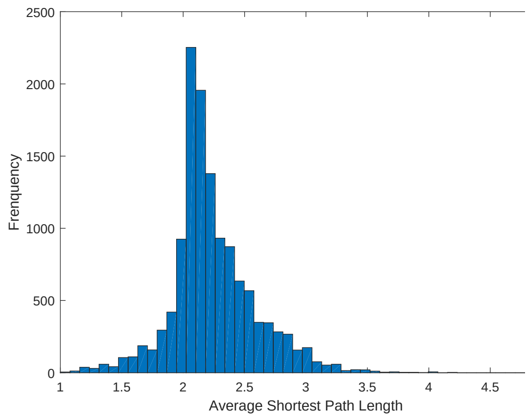
(b) Density



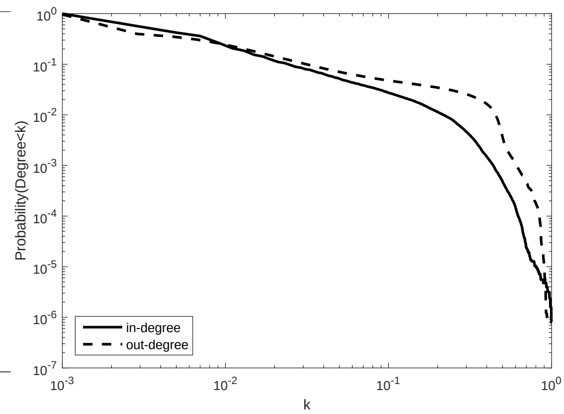
(c) Assortativity between in-degrees



(d) Assortativity between out-degrees



(e) Average shortest path



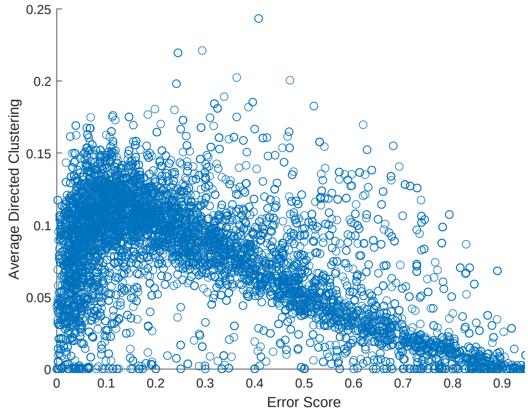
(f) Degree distribution

Figure 4: Histograms of key network measures

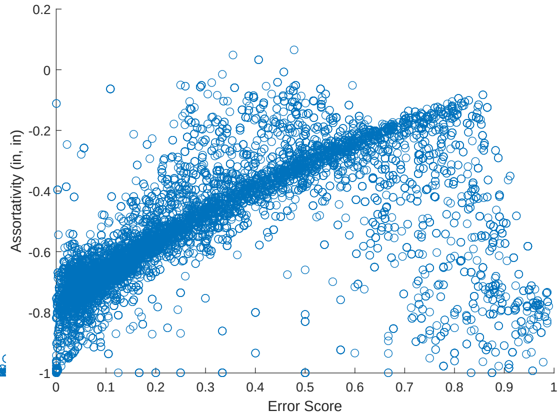
This table shows the distribution of the variables in our model split by core and periphery banks. Order denotes the position at which the bank was chosen to enter the market (relative to the number of banks in the system), Shock denotes the difference between the optimal and actual cash reserves, Utility the ratio of the initial and final utility levels, leverage the ratio of the initial and final leverage, Borrowing (Lending) denotes the amount of interbank borrowing (lending) relative to the total assets and Net Borrow the difference between these, the Borrow (Lending) rate denotes the spread between the average of a bank's interbank borrowing (lending) rate and the risk free rate, and the spread its difference, Reserves denotes the ratio of the final and initial cash reserve ratios, Return denotes the ratio of the final and initial return on equity, EV central denotes the eigenvector centrality and Clustering the directed clustering coefficient.

Variable	Position	Minimum	1% quantile	25% quantile	Median	75% quantile	99% quantile	Maximum
Order	Core	0.0010	0.0011	0.09935	0.0300	0.1032	0.8925	1.0000
	Periphery	0.0010	0.0208	0.2556	0.5000	0.7496	0.9909	1.0000
Shock	Core	-81.1780	-32.8400	-10.3490	-3.5097	5.7558	44.6090	89.3330
	Periphery	-72.8410	-28.7170	-8.5712	-2.6207	8.2208	41.6220	90.4460
Utility	Core	1.0000	1.0009	1.3021	1.1213	1.4967	21.228	548.7400
	Periphery	1.0000	1.0000	1.0000	1.0006	1.0037	1.0224	9.5645
Leverage	Core	1.0000	1.0000	1.1691	1.4303	2.4465	8.6173	16.9120
	Periphery	1.0000	1.0000	1.0000	1.0199	1.0696	1.2265	3.5154
Borrowing	Core	0.0000	0.0000	0.1446	0.3009	0.5913	0.8840	0.9409
	Periphery	0.0000	0.0000	0.0000	0.1952	0.0651	0.1847	0.7155
Lending	Core	0.0000	0.0000	0.1257	0.2963	0.5464	0.8445	0.9424
	Periphery	0.0000	0.0000	0.0000	0.0000	0.0551	0.4121	0.9250
Net Borrow	Core	-0.8968	-0.3496	-0.0056	0.0393	0.0860	0.2745	0.6886
	Periphery	-0.9250	-0.4120	-0.0546	0.0192	0.0646	0.1839	0.3373
Borrow rate	Core	0.0001	0.0055	0.0478	0.0794	0.1241	1.1800	9.1866
	Periphery	0.0006	0.0145	0.1063	0.1887	0.3890	8.1960	621.9400
Lending rate	Core	0.0011	0.0133	0.1020	0.1708	0.2903	5.2451	1676.3000
	Periphery	0.0000	0.0064	0.0453	0.0757	0.1116	0.2576	3343.8000
Spread	Core	-1674.9410	-5.2690	-0.2030	-0.0739	-0.0208	0.7156	8.9034
	Periphery	-3343.3450	-1.2402	-0.0936	-0.0229	0.0240	3.7213	58.8202
Reserves	Core	0.0000	0.0000	0.0889	0.4245	1.0510	4.8145	167.3000
	Periphery	0.0000	0.0000	0.2081	0.7290	1.2833	6.2587	7608.8000
Return	Core	-1.1561	0.4246	1.015	1.2219	2.0025	35.2640	541.8500
	Periphery	-3.6625	0.3376	0.9022	0.9783	1.0279	1.2174	42.4270
EV Central	Core	0.0001	0.0014	0.0039	0.0082	0.0163	0.0813	0.4142
	Periphery	0.0000	0.0000	0.0009	0.0016	0.0024	0.0132	0.3333
Clustering	Core	0.0000	0.0000	0.0052	0.0175	0.0483	0.2411	0.5278
	Periphery	0.0000	0.0000	0.0000	0.0000	0.1944	0.5833	0.9167

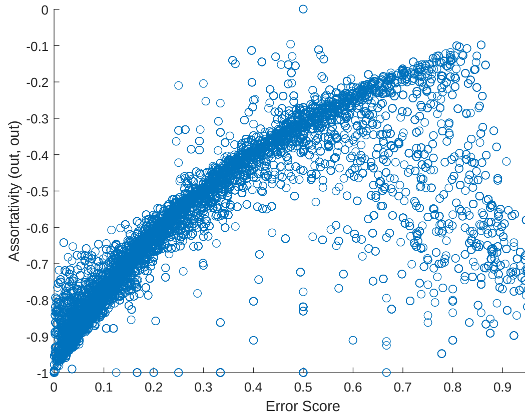
Table 3: Descriptive statistics of variables in the model split by core and periphery banks



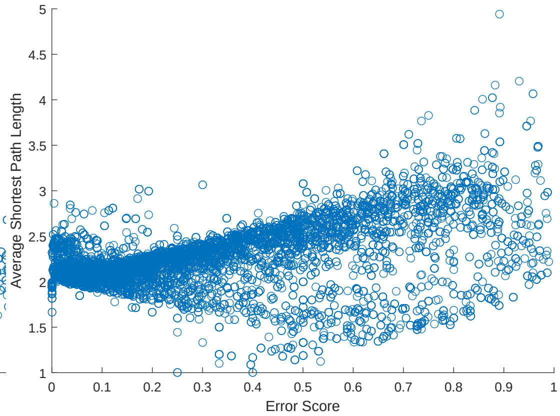
(a) Clustering coefficient



(b) Assortativity in-degree



(c) Assortativity out-degree



(d) Average shortest path length

Figure 5: Scatter plots of network measures against the error score

banks lending to them. The interest rates between banks in the core and periphery, respectively, are falling between these extremes.

We also clearly can see from table 3 that banks in the core are much more active in the interbank market than banks in the periphery as evidenced by their higher interbank borrowing and lending as well as higher leverage. At the same time we observe that banks in the core hold fewer cash reserves than banks in the periphery, suggesting the increased leverage does not result in an equally increased cash reserve but instead banks seeking a higher return. Not surprisingly, therefore, banks in the core show a higher return on equity and an improved utility level. It is thus that being in the core is beneficial, but it is also clear that banks selected to participate in the interbank market early, are more likely to end up in the core.

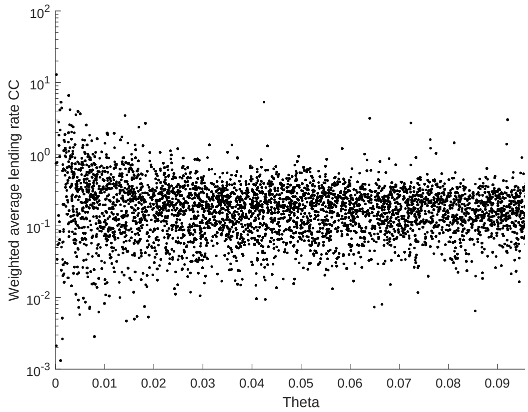
This table shows the estimates of a OLS regression to study the emergence of core periphery structure. The three dependent variables describe or relate to the core periphery structure, which are ERROR SCORE, CORE SIZE and CORE CENTRALITY. The independent variables are all the exogenous parameters in our model. We show estimates of these regressions and do not report t-values as due to the sample size of 10,000 all estimates are statistically significant. We report the R^2 statistics for each regression.

	log(Error score)	log(Core size)	log(Core centrality)
Constant	2.4514	2.1173	−1.1739
$\log(\theta)$	0.8583	0.1103	0.0338
$\log(Q)$	−0.2949	−0.6082	0.3558
$\log(N)$	−0.2909	−0.8503	−0.4926
$\frac{1}{\lambda}$	3.8407	−0.6581	0.8731
β	0.0481	−0.0685	0.0356
r^f	12.1977	0.4171	1.2962
r^D	−1.9401	−0.8948	−0.2026
r^C	−8.9183	−0.6399	−0.8429
R^2	0.7502	0.8648	0.8310

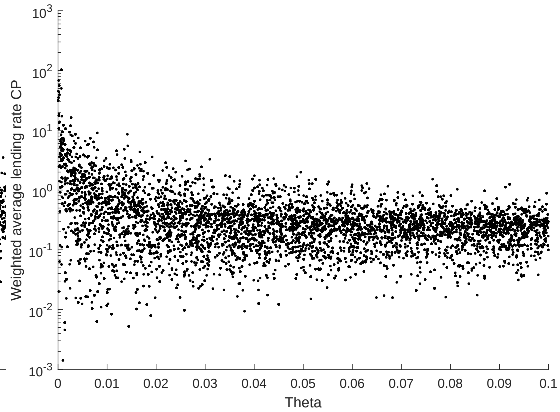
Table 4: OLS regression for the determinants of core periphery structure

Given the large number of free parameters in our model, we also conducted a regression analysis of the results as detailed in table 4. Given the sample size of 10,000 computer experiments, the statistical significance of the estimates are not meaningful. [1] We see that the core-periphery structure becomes more pronounced, i. e. a lower error score is shown, if banks put less emphasis on liquidity concerns (θ) as being active in the interbank market is a prerequisite for the emergence of a core. Focusing on profitability makes banks borrow and lend repeatedly and thus establish themselves in the core. The implied emphasis on profitability in banks with a higher leverage (λ) has a similar effect, although it increases rather than reduces the core size due to the market being dominated by fewer banks and increases its centrality. More banks (N) enable the establishment of a core more easily as it allows for more transactions between banks, facilitating for core banks to emerge. This core is then smaller, relative to the number of banks, however this comes at the cost of the centrality of this core. Using larger interbank loans (Q) leads to better core-periphery structures as the lower number of transactions will be more concentrated and thus

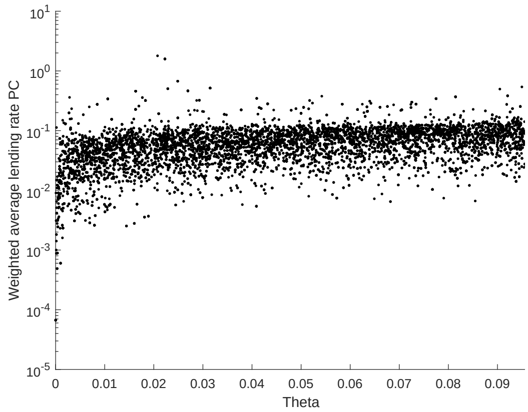
[1] We note, however, that the R^2 in all regressions is considerable, indicating a good fit of the regression overall. As all explanatory variables are chosen exogenously we can discard the problem of endogeneity and in Appendix B show that our results are robust for multicollineraity.



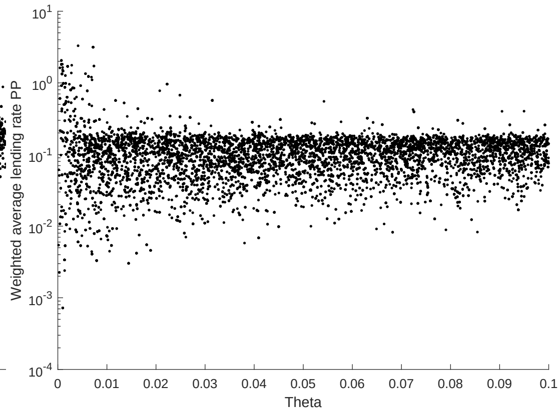
(a) Core-Core lending rates



(b) Core-Periphery lending rates



(c) Periphery-Core lending rates



(d) Periphery-Periphery lending rates

Figure 6: Dependence of the interbank lending rate on θ

allowing for less lending between banks in the periphery. This also reduces the size of the core while increasing its centrality. The impact of the interest rates are such that they make the focus of the banks on profitability more or less pronounced and thus affect the extent banks engage in interbank lending and borrowing, influencing the emergence of a core.

Furthermore, lending rates by banks in the core are lower if banks have stronger preferences for cash. This relationship is only strong for very low values of θ, though. The opposite effect can be observed for banks in the periphery, it is however much more restricted to very low values of θ as we can see from figure 6.

6 Banks' positions within the interbank lending network

After having established that the structure of the interbank network shows clear signs of a core-periphery structure with these groups of banks exhibiting clearly different properties, we can now also assess, based on the model developed above as well as the simulations, how individual banks are affected. Our analysis is summarized in figure 7. We start with the only two exogenous variables in our model that distinguish banks from each other, the initial liquidity shock and the timing at which the bank enters the interbank market. It is relatively straightforward to see that the larger this liquidity shock, the deviation from the optimal cash reserves, the more banks are willing to lend due to excess cash reserves and the less they are willing to borrow. The opposite will be true for negative liquidity shocks. Therefore, larger liquidity shocks will generally induce a bank to become more active in the interbank market. A bank entering the interbank market early will have more opportunities to be engaged in trading as more banks subsequently enter the market. Therefore increased participation in the interbank market makes it more likely that a bank has many connections with other banks and is thus more likely to be found to be in the core.

The cash reserves are increased from lending and reduced from borrowing. With an increase in the amount of borrowing and lending, the overall incentives are such that the cash reserves are falling, with banks having a clear incentive to use their lower reservation prices to lend some of the cash and generate additional profits that increase their utility level, partly offsetting the resulting lower cash reserves through additional borrowing at even lower rates. Any increase in borrowing will obviously increase the leverage of the bank. As we have seen in the discussion of the model, banks with higher leverage have lower reservation prices for interbank lending and borrowing, and the difference between them reduces.

Increased borrowing and lending will usually mean that banks can make higher prof-

This table shows regression result of the following models. Model (1) and (2) use probit regression on dependent variable ISCORE, which is a dummy variable of whether the bank is in core. The difference between (1) and (2) is that (1) use sample where shocks are negative (surplus of liquidity) while (2) use sample where shocks are positive (shortage of liquidity). We show probit estimates of these regressions together with the Pseudo-R^2. In the row for R^2, we show McFadden's R^2. Model (3) and (4) use OLS regression on dependent variable NET LENDING, which is a bank's interbank lending minus interbank borrowing normalized by its total assets. Again, we split sample and use sample with negative shocks in (3) and positive shocks in (4). Model (5) and (6) use OLS regression on dependent variable FINAL LEVERAGE, which is a bank's asset over equity at the end. We use sample with negative shock in (5) and positive in (6). For model (3)-(6), we report OLS estimates with R^2 values and do not report t-values as due to the sample size of 10,000 all estimates are statistically significant.

	Prob(Core)		Net lending		Final leverage	
	Positive	Negative	Positive	Negative	Positive	Negative
	(1)	(2)	Positive	Negative	Positive	Negative
Constant	−0.8878	−0.8624	−0.0193	−0.0077	16.0092	11.0600
Order	−0.6580	−0.9051	0.0007	−0.0023	−0.1714	−0.1058
Size of shock	0.9232	0.6140	−0.4130	0.7255	−30.7669	−7.6775
R^2	0.2794	0.3183	0.7373	0.7954	0.0300	0.0061

Table 5: Regression of banks' probability in core, net lending and leverage.

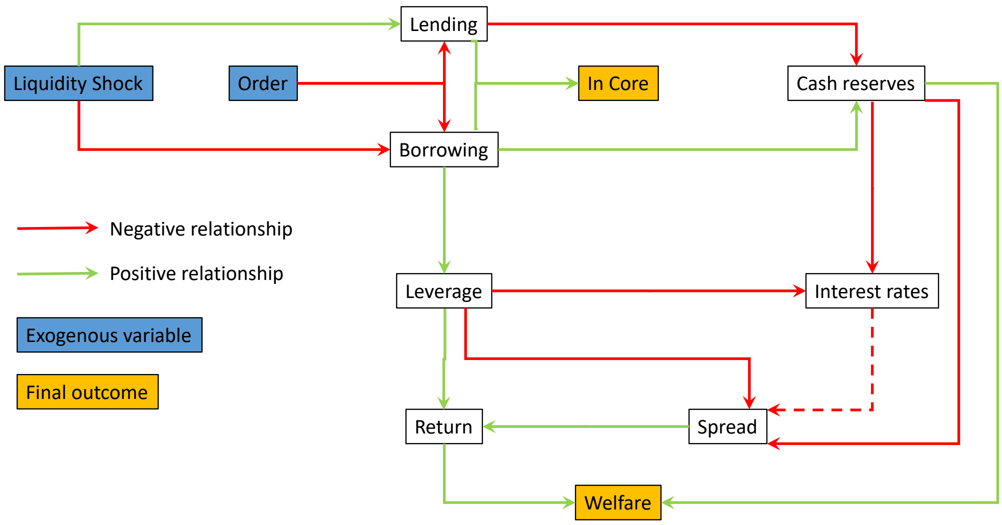


Figure 7: Relationships between variables

its from their borrowing and lending, more than offsetting the effect of a smaller spread. With core banks lending mostly to periphery banks that have higher reservation prices and lower reservation prices for borrowing due to the larger spread as a result of their lower leverage, they are able to maintain their profit margins and thus generate larger returns. The result is that the higher returns more than offset the lower levels of cash reserves, thus increasing the overall level of utility of banks in the core.

As indicated some of these results can be derived directly from the model, others can be derived from the computer experiments. We show a few selected regression results using our only exogenous variables in table 5. As we can see from the descriptive statistics above, some results show clear outliers and we have addressed these by also investigating robust regressions, quantile regressions, winsorizing, as well as trimming of data and found the results to be robust. We clearly see the importance of the order in which banks enter the market (Order) for them being in the core and having a high final leverage. Similarly, a large shock, either negative or positive, increases the likelihood of being in the core due to the amount of transaction such banks engage him to offset this shock, but reduces the final leverage as banks are more concerned about offsetting this shock than borrowing and lending in equal parts. We can see this from the impact on net lending where the size of the shock is most important but the order of market entry has no meaningful impact.

7 Conclusions

We developed a model of interbank lending that balanced the liquidity and profitability concerns of banks. Deriving reservation prices for borrowing and lending as well as their properties we then showed how the bilateral interactions between banks gave rise to a core-periphery structure of the resulting interbank network, besides other realistic properties. Finally we considered the properties of banks in the core and periphery as well as the determinants of a bank's position in these. In doing so we noted that core banks tend to lend at higher and borrow at lower rates than banks in the periphery. Furthermore, banks that enter the market early or face a larger liquidity shock are most likely to become core banks.

By deriving realistic properties of the interbank lending network in the absence of credit risk we can show that the observed properties are the result primarily of bilateral transactions between banks rather than any differences in their ex-ante properties. As Cohen-Cole et al. (2010), Hałaj & Kok (2015), Lux (2015), and Castiglionesi & Navarro (2016) showed the emergence of core-periphery structures in the presence of counterparty risk and Farboodi (2014) and in't Veld et al. (2014) for heterogeneous banks, our results suggest this to be a universal property of interbank lending markets.

The model presented here is very minimal in that it only considers banks differing in an exogenous liquidity shock and the time of their market entry. We do not consider the credit risk arising in such a setting; it is obvious that any bank in the core with its higher leverage would have a higher credit risk. We leave the exploration of this aspect for future research. Also, the interbank market cannot be seen in isolation from the monetary operations of central banks that will supply or withdraw liquidity in the banking system. How such operations affect interbank lending is beyond the scope of this paper and therefore left for future consideration.

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A Appendix: Proofs

A.1 Proposition 1

The cash level after any transaction must be non-negative, i.e.

$$\mathbf{R}_i + \Delta\mathbf{R}_i = \Delta\mathbf{B}_i - \Delta\mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i \geq 0,$$

where $\Delta\mathbf{L}_i \geq 0$ and $\Delta\mathbf{B}_i \geq 0$. Hence

$$\begin{aligned}
\rho_i(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i) &= \frac{\mathbf{R}_i + \Delta\mathbf{B}_i - \Delta\mathbf{L}_i}{\mathbf{D}_i + \mathbf{B}_i + \Delta\mathbf{B}_i}, \\
r_i^E(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i) &= r_{i,0}^E + \frac{(r_i^L - r^f)\Delta\mathbf{L}_i + (r^f - r_i^B)\Delta\mathbf{B}_i}{\mathbf{E}_i},
\end{aligned}$$

where $r_{i,0}^E$ denotes the return on equity prior to the transaction. The utility maximizing problem of bank i is thus

$$\begin{aligned}
\max_{\Delta\mathbf{B}_i, \Delta\mathbf{L}_i} \quad & U_i(\rho_i(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i), r_i^E(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i)) \\
\text{subject to} \quad & \Delta\mathbf{B}_i \geq 0, \\
& \Delta\mathbf{L}_i \geq 0, \\
& \Delta\mathbf{B}_i - \Delta\mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i \geq 0.
\end{aligned}$$

To prove this proposition, we need to show that the above problem has a unique solution. The Lagrangian of this optimization problem is

$$\begin{aligned}
\mathcal{L} &= U_i(\rho_i(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i), r_i^E(\Delta\mathbf{B}_i, \Delta\mathbf{L}_i)) \\
& \quad + \lambda_1 \Delta\mathbf{B}_i + \lambda_2 \Delta\mathbf{L}_i + \lambda_3(\Delta\mathbf{B}_i - \Delta\mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i)
\end{aligned}$$

with Kuhn-Tucker conditions

$$\begin{aligned}
\frac{\partial U_i}{\partial \Delta \mathbf{B}_i} + \lambda_1 + \lambda_3 &= 0, \\
\frac{\partial U_i}{\partial \Delta \mathbf{L}_i} + \lambda_2 - \lambda_3 &= 0, \\
\Delta \mathbf{B}_i &\geq 0, \\
\lambda_1 &\geq 0, \\
\lambda_1 \Delta \mathbf{B}_i &= 0, \\
\Delta \mathbf{L}_i &\geq 0, \\
\lambda_2 &\geq 0, \\
\lambda_2 \Delta \mathbf{L}_i &= 0, \\
\Delta \mathbf{B}_i - \Delta \mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i &\geq 0, \\
\lambda_3 &\geq 0, \\
\lambda_3(\Delta \mathbf{B}_i - \Delta \mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i) &= 0.
\end{aligned}$$

Adding the first two conditions gives us

$$\frac{\partial U_i}{\partial \Delta \mathbf{B}_i} + \frac{\partial U_i}{\partial \Delta \mathbf{L}_i} + \lambda_1 + \lambda_2 = 0.$$

Given that

$$\begin{aligned}
\frac{\partial U_i}{\partial \Delta \mathbf{B}_i} &= \frac{\partial U_i}{\partial \rho_i} \frac{\mathbf{L}_i + \mathbf{C}_i - \mathbf{E}_i + \Delta \mathbf{L}_i}{(\mathbf{D}_i + \mathbf{B}_i + \Delta \mathbf{B}_i)^2} + \frac{\partial U_i}{\partial r_i^E} \frac{r^f - r^B}{\mathbf{E}_i}, \\
\frac{\partial U_i}{\partial \Delta \mathbf{L}_i} &= -\frac{\partial U_i}{\partial \rho_i} \frac{\mathbf{B}_i + \Delta \mathbf{B}_i + \mathbf{D}_i}{\mathbf{D}_i + \mathbf{B}_i + \Delta \mathbf{B}_i} + \frac{\partial U_i}{\partial r_i^E} \frac{r^L - r^f}{\mathbf{E}_i}.
\end{aligned}$$

it is

$$\begin{aligned}
\frac{\partial U_i}{\partial \Delta \mathbf{B}_i} + \frac{\partial U_i}{\partial \Delta \mathbf{L}_i} &= -\frac{\partial U_i}{\partial \rho_i} \frac{\Delta \mathbf{B}_i - \Delta \mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i}{(\mathbf{D}_i + \mathbf{B}_i + \Delta \mathbf{B}_i)^2} \\
&\quad + \frac{\partial U_i}{\partial r_i^E} \frac{r_i^L - r_i^B}{\mathbf{E}_i} < 0.
\end{aligned}$$

as $\Delta \mathbf{B}_i - \Delta \mathbf{L}_i + \mathbf{E}_i + \mathbf{D}_i + \mathbf{B}_i - \mathbf{C}_i - \mathbf{L}_i \geq 0$ and $r_i^L - r_i^B < 0$ arising from the non-arbitrage interbank market rates as shown below.

Combining these results we find that $\lambda_1 + \lambda_2 \geq 0$. Using the other Kuhn-Tucker conditions we can distinguish between three cases:

1. If $\lambda_1 > 0$ and $\lambda_2 > 0$ we require that $\Delta\mathbf{B}_i^* = \Delta\mathbf{L}_i^* = 0$, which is a unique solution. This corresponds to the case that bank i's interbank positions are already optimal.

2. If $\lambda_1 > 0$ and $\lambda_2 = 0$ we find that $\Delta\mathbf{B}_i^* = 0$ and $\Delta\mathbf{L}_i^*$ solves for $\frac{\partial U_i}{\partial \Delta\mathbf{L}_i}(\Delta\mathbf{L}_i^*) = 0$. Such $\Delta\mathbf{L}_i^*$ must exist since the second constraint is non binding and it is unique as the utility function is concave in $\Delta\mathbf{L}_i^*$. This corresponds to the case that bank i is to increase interbank lending.

3. If $\lambda_1 = 0$ and $\lambda_2 > 0$ we find that $\Delta\mathbf{L}_i^* = 0$ and $\Delta\mathbf{B}_i^*$ solves for $\frac{\partial U_i}{\partial \Delta\mathbf{B}_i}(\Delta\mathbf{B}_i^*) = 0$. Such $\Delta\mathbf{B}_i^*$ must exist since the first constraint is non binding and it is unique as the utility function is concave of $\Delta\mathbf{B}_i^*$. This corresponds to the case that bank i is to increase interbank borrowing.

The last step is to show the utility function is concave in $\Delta\mathbf{B}_i$ and $\Delta\mathbf{L}_i$. We need only to check the second derivatives are negative, $\frac{\partial^2 U_i}{\partial \Delta\mathbf{B}_i^2} < 0$ and $\frac{\partial^2 U_i}{\partial \Delta\mathbf{L}_i^2} < 0$, as the first derivatives are trivially shown to be positive.

We find that

$$
\begin{aligned}
\frac{\partial^2 U_i}{\partial \Delta\mathbf{B}_i^2} &= U_1 \frac{\partial^2 \rho_i}{\partial \Delta\mathbf{B}_i^2} + U_2 \frac{\partial^2 r_i^{\mathbf{E}}}{\partial \Delta\mathbf{B}_i^2} + U_{11}\left(\frac{\partial \rho_i}{\partial \Delta\mathbf{B}_i}\right)^2 \\
&\quad + 2U_{12}\frac{\partial \rho_i}{\partial \Delta\mathbf{B}_i}\frac{\partial r_i^E}{\partial \Delta\mathbf{B}_i} + U_{22}\left(\frac{\partial r_i^E}{\partial \Delta\mathbf{B}_i}\right)^2, \\
\frac{\partial^2 U_i}{\partial \Delta\mathbf{L}_i^2} &= U_1 \frac{\partial^2 \rho_i}{\partial \Delta\mathbf{L}_i^2} + U_2 \frac{\partial^2 r_i^E}{\partial \Delta\mathbf{L}_i^2} + U_{11}\left(\frac{\partial \rho_i}{\partial \Delta\mathbf{L}_i}\right)^2 \\
&\quad + 2U_{12}\frac{\partial \rho_i}{\partial \Delta\mathbf{L}_i}\frac{\partial r_i^E}{\partial \Delta\mathbf{L}_i} + U_{22}\left(\frac{\partial r_i^E}{\partial \Delta\mathbf{L}_i}\right)^2,
\end{aligned}
$$

which are both negative as

$$
\begin{aligned}
U_1 &= \frac{\partial U_i}{\partial \rho_i} \geq 0, \\
U_2 &= \frac{\partial U_i}{\partial r_i^E} \geq 0, \\
U_{12} &= \frac{\partial^2 U_i}{\partial \rho \partial r_i^E} \leq 0, \\
U_{11} &= \frac{\partial^2 U_i}{\partial \rho^2} \leq 0, \\
U_{22} &= \frac{\partial^2 U_i}{\partial r_i^{E\ 2}} \leq 0, \\
\frac{\partial \rho_i}{\partial \Delta \mathbf{B}_i} &= \frac{\mathbf{L}_i + \Delta \mathbf{L}_i + \mathbf{C}_i - \mathbf{E}_i}{(\mathbf{D}_i + \mathbf{B}_i + \Delta \mathbf{B}_i)^2} \geq 0, \\
\frac{\partial^2 \rho_i}{\partial \Delta \mathbf{B}_i^2} &= -2\frac{\mathbf{L}_i + \Delta \mathbf{L}_i + \mathbf{C}_i - \mathbf{E}_i}{(\mathbf{D}_i + \mathbf{B}_i + \Delta \mathbf{B}_i)^3} \leq 0, \\
\frac{\partial r^E}{\partial \Delta \mathbf{B}_i} &= \frac{r^f - r_i^B}{\mathbf{E}_i} \leq 0, \\
\frac{\partial^2 r_i^E}{\partial \Delta \mathbf{B}_i^2} &= 0, \\
\frac{\partial \rho_i}{\partial \Delta \mathbf{L}_i} &= -\frac{1}{\mathbf{D}_i + \mathbf{B}_i} \leq 0, \\
\frac{\partial^2 \rho_i}{\partial \Delta \mathbf{L}_i^2} &= 0, \\
\frac{\partial r^E}{\partial \Delta \mathbf{L}_i} &= \frac{r_i^L - r^f}{\mathbf{E}_i} \geq 0, \\
\frac{\partial^2 r^E}{\partial \Delta \mathbf{L}_i^2} &= 0.
\end{aligned}
$$

A.2 Proposition 2

To determine the reservation prices of a borrowing or lending, we require that the utilities before and after a transaction are identical, i. e. $U_i(\rho_i, r_i^E) = U_i(\rho_{i,B}, r_{i,B}^E)$ and $U_i(\rho_i, r_i^E) = U_i(\rho_{i,L}, r_{i,L}^E)$, where the subscript B and L denote borrowing and lending, respectively.

Consider bank i, whose current cash ratio and return on equity are given by

$$
\begin{aligned}
\rho_i &= \frac{\mathbf{R}_i}{\mathbf{D}_i + \mathbf{B}_i}, \\
r_i^E &= \frac{r^C \mathbf{C}_i + r_i^L \mathbf{L}_i + r^f \mathbf{R}_i - r^D \mathbf{D}_i - r_i^B \mathbf{B}_i}{\mathbf{E}_i}.
\end{aligned}
$$

After a transaction these variables become

$$
\begin{aligned}
\rho_{i,B} &= \frac{\mathbf{R}_i + Q}{\mathbf{D}_i + \mathbf{B}_i + Q}, \\
\rho_{i,L} &= \frac{\mathbf{R}_i - Q}{\mathbf{D}_i + \mathbf{B}_i}, \\
r_{i,B}^E &= r_i^E + \frac{Q(r^f - r_i^a)}{\mathbf{E}_i}, \\
r_{i,L}^E &= r_i^E + \frac{Q(r_i^b - r^f)}{\mathbf{E}_i}.
\end{aligned}
$$

Inserting these into the utility function we obtain for interbank borrowing

$$
\gamma_i \left(\frac{\mathbf{R}_i}{\mathbf{D}_i + \mathbf{B}_i} \right)^{\theta_i} \left(1 + r_i^E\right)^{1-\theta_i} = \gamma_i \left(\frac{\mathbf{R}_i + Q}{\mathbf{D}_i + \mathbf{B}_i + Q} \right)^{\theta_i} \left(1 + r_i^E + \frac{Q(r^f - r_i^a)}{\mathbf{E}_i} \right)^{1-\theta_i}.
$$

which can easily be seen to solve for

$$
r_i^a - r^f = \quad 1 - \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q} \left(1 + \frac{Q}{\mathbf{D}_i + \mathbf{B}_i} \right) \right)^{\frac{\theta_i}{1-\theta_i}} \Bigg) \frac{\mathbf{E}_i \left(1 + r_i^E\right)}{Q}.
$$

Similarly for interbank lending we obtain

$$
\gamma_i \left(\frac{\mathbf{R}_i}{\mathbf{D}_i + \mathbf{B}_i} \right)^{\theta_i} \left(1 + r_i^E\right)^{1-\theta_i} = \gamma_i \left(\frac{\mathbf{R}_i - Q}{\mathbf{D}_i + \mathbf{B}_i} \right)^{\theta_i} \quad 1 + r_i^E + \frac{Q\left(r_i^b - r^f\right)}{\mathbf{E}_i} \Bigg)^{1-\theta_i},
$$

which solves for

$$
r_i^b - r^f = \left(\left(\frac{\mathbf{R}_i}{\mathbf{R}_i - Q} \right)^{\frac{\theta_i}{1-\theta_i}} - 1 \right) \frac{\mathbf{E}_i \left(1 + r_i^E\right)}{Q}.
$$

which gives the rate for interbank lending.

A.3 Lemma 1

Since $\mathbf{B}_i = \mathbf{L}_i = 0$, the optimal cash level is found by solving the optimization problem

$$
\begin{aligned}
&\max_{\mathbf{R}_i} && U_i\left(\rho_i, r_i^E\right) \\
&\text{subject to} && \mathbf{R}_i + \mathbf{C}_i = \mathbf{D}_i + \mathbf{E}_i = \mathbf{A}_i.
\end{aligned}
$$

Solving this optimization problem we obtain the optimal amount of cash as

$$R_i^* = \theta_i \frac{\mathbf{E}_i \left(1 + r_i^C\right) + \mathbf{D}_i \left(r_i^C - r_i^D\right)}{r_i^C - r^f},$$

and therefore the optimal liquidity ratio is

$$\rho_i^* = \frac{\mathbf{R}_i^*}{\mathbf{D}_i} = \theta_i \frac{\frac{\mathbf{E}_i}{\mathbf{D}_i}\left(1 + r_i^C\right) + \left(r_i^C - r_i^D\right)}{r_i^C - r^f}.$$

Since $\lambda_i = \frac{\mathbf{A}_i}{\mathbf{E}_i}$, we have

$$\frac{\mathbf{E}_i}{\mathbf{D}_i} = \frac{\mathbf{E}_i}{\mathbf{A}_i - \mathbf{E}_i} = \frac{1}{\frac{\mathbf{A}_i}{\mathbf{E}_i} - 1} = \frac{1}{\lambda_i - 1}$$

and thus

$$\rho_i^* = \theta_i \frac{\frac{1}{\lambda_i - 1}(r_i^C + 1) + (r_i^C - r_i^D)}{r_i^C - r^f},$$

from which we easily see that

$$\frac{\partial \rho_i^*}{\partial \lambda_i} = -\theta_i \frac{1 + r_i^C}{r_i^C - r^f} \frac{1}{(\lambda_i - 1)^2} < 0.$$

Similarly we obtain that

$$\begin{aligned}
\frac{\partial \rho_i^*}{\partial r_i^D} &= -\frac{\theta_i}{r_i^C - r^f} < 0 \\
\frac{\partial \rho_i^*}{\partial r_i^C} &= \frac{\theta_i}{(r_i^C - r^f)^2} \left(r_i^D - r^f - \frac{E_i}{D_i}(1 + r^f) \right) < 0 \\
\frac{\partial \rho_i^*}{\partial r^f} &= \frac{\theta_i}{(r_i^C - r^f)^2} > 0
\end{aligned}$$

A.4 Lemma 2

Denote $q_i = \frac{Q}{\mathbf{R}_i}$ and $\eta_i = \frac{Q}{\mathbf{D}_i + \mathbf{B}_i}$ and the proof reduces to showing that $0 < 1 - \left(\frac{1}{1+q_i}(1 + q_i\eta_i)\right)^{\frac{\theta_i}{1-\theta_i}} < \left(\frac{1}{1-q_i}\right)^{\frac{\theta_i}{1-\theta_i}} - 1$.

Since $\frac{\theta_i}{1-\theta_i} > 0$, the first inequality reduces to $1 > \frac{1}{1+q_i}(1 + q_i\eta_i)$ which is trivially fulfilled as $\eta_i < 1$.

The second inequality is equivalent to $2 < \left(\frac{1}{1+q_i}(1 + q_i\eta_i)\right)^{\frac{\theta_i}{1-\theta_i}} + \left(\frac{1}{1-q_i}\right)^{\frac{\theta_i}{1-\theta_i}}$. With $0 < \eta_i < 1$ and $0 < q_i < 1$ this can easily be shown to reduce to $0 < \left((1 + q_i)^{\frac{\theta_i}{2(1-\theta_i)}} - (1 - q_i)^{\frac{\theta_i}{2(1-\theta_i)}}\right)^2$, which obviously is not negative.

A.5 Lemma 3

To simplify notation, we introduce the following definitions:

$$
\begin{aligned}
f_i &= \frac{\mathbf{E}_i(1+r_i^E)}{Q} > 0, \\
g_i^a &= 1 - \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q} \left(1 + \frac{Q}{\mathbf{D}_i + \mathbf{B}_i} \right) \right)^{\frac{\theta_i}{1-\theta_i}} > 0, \\
g_i^b &= \left(\frac{\mathbf{R}_i}{\mathbf{R}_i - Q} \right)^{\frac{\theta_i}{1-\theta_i}} - 1 > 0,
\end{aligned}
$$

so that we can write r_i^a and r_i^b as

$$
\begin{aligned}
r_i^a &= r^f + g_i^a f_i, \\
r_i^b &= r^f + g_i^b f_i.
\end{aligned}
$$

Taking partial derivatives we obtain

$$
\begin{aligned}
\frac{\partial r_i^a}{\partial \rho_i} &= g_i^a \frac{\partial f_i}{\partial \rho_i} + \frac{\partial g_i^a}{\partial \rho_i} f_i, \\
\frac{\partial r_i^b}{\partial \rho_i} &= g_i^b \frac{\partial f_i}{\partial \rho_i} + \frac{\partial g_i^b}{\partial \rho_i} f_i, \\
\frac{\partial r_i^a}{\partial \lambda_i} &= g_i^a \frac{\partial f_i}{\partial \lambda_i} + \frac{\partial g_i^a}{\partial \lambda_i} f_i, \\
\frac{\partial r_i^b}{\partial \lambda_i} &= g_i^b \frac{\partial f_i}{\partial \lambda_i} + \frac{\partial g_i^b}{\partial \lambda_i} f_i.
\end{aligned}
$$

We can now easily show that with $r_i^{B,D} = \frac{\mathbf{D}_i r_i^D + \mathbf{B}_i r_i^B}{\mathbf{D}_i + \mathbf{B}_i}$ and $r_i^{C,L} = \frac{\mathbf{C}_i r_i^C + \mathbf{L}_i r_i^L}{\mathbf{C}_i + \mathbf{L}_i}$

$$
\begin{aligned}
\frac{\partial f_i}{\partial \lambda_i} &= \frac{\mathbf{A}_i}{Q}\left(-1 + \rho_i(r_i^f - r_i^{C,L}) - r_i^{B,D}\right)\frac{1}{\lambda_i^2} < 0, \\
\frac{\partial f_i}{\partial \rho_i} &= \frac{\mathbf{A}_i}{Q}\left(1 - \frac{1}{\lambda_i}\right)\left(r_i^f - r_i^{C,L}\right) < 0, \\
\frac{\partial g_i^a}{\partial \lambda_i} &= -\frac{\theta_i}{1-\theta_i}\left(\frac{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + Q}{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + \frac{Q}{\rho_i}}\right)^{\frac{\theta_i}{1-\theta_i}-1} \frac{\frac{1}{\lambda_i^2}\mathbf{A}_i Q\left(\frac{1}{\rho_i} - 1\right)}{\left(\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + \frac{Q}{\rho_i}\right)^2} < 0, \\
\frac{\partial g_i^a}{\partial \rho_i} &= -\frac{\theta_i}{1-\theta_i}\left(\frac{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + Q}{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + \frac{Q}{\rho_i}}\right)^{\frac{\theta_i}{1-\theta_i}} \frac{Q}{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i\rho_i^2 + \rho_i Q} < 0, \\
\frac{\partial g_i^b}{\partial \lambda_i} &= -\frac{\theta_i}{1-\theta_i}\left.\frac{\mathbf{A}_i}{\mathbf{A}_i - \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}}\right)^{\frac{\theta_i}{1-\theta_i}} \frac{\frac{Q}{\rho_i(\lambda_i-1)^2}}{\mathbf{A}_i - \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}} < 0, \\
\frac{\partial g_i^b}{\partial \rho_i} &= -\frac{\theta_i}{1-\theta_i}\left.\frac{\mathbf{A}_i}{\mathbf{A}_i - \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}}\right)^{\frac{\theta_i}{1-\theta_i}} \frac{\frac{Q}{\rho_i^2}\frac{\lambda_i}{\lambda_i-1}}{\mathbf{A}_i - \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}} < 0.
\end{aligned}
$$

The negative signs on all these partial derivatives proves the negative signs in the lemma.

Furthermore we get

$$
\begin{aligned}
\frac{\partial\left(r_i^b - r_i^a\right)}{\partial \rho_i} &= \left(g_i^b - g_i^a\right)\frac{\partial f_i}{\partial \rho_i} + \left(\frac{\partial g_i^b}{\partial \rho_i} - \frac{\partial g_i^a}{\partial \rho_i}\right) f_i, \\
\frac{\partial\left(r_i^b - r_i^a\right)}{\partial \lambda_i} &= \left(g_i^b - g_i^a\right)\frac{\partial f_i}{\partial \lambda_i} + \left(\frac{\partial g_i^b}{\partial \lambda_i} - \frac{\partial g_i^a}{\partial \lambda_i}\right) f_i.
\end{aligned}
$$

Knowing that $\frac{\partial f_i}{\partial \rho_i} < 0$, $\frac{\partial f_i}{\partial \lambda_i} < 0$, $f_i > 0$, and $g_i^b - g_i^a > 0$, it remains to be shown that $\frac{\partial g_i^b}{\partial \rho_i} - \frac{\partial g_i^a}{\partial \rho_i} < 0$ and $\frac{\partial g_i^b}{\partial \lambda_i} - \frac{\partial g_i^a}{\partial \lambda_i} < 0$. Using previous results we easily obtain

$$
\begin{aligned}
\frac{\frac{\partial g_i^b}{\partial \rho_i}}{\frac{\partial g_i^a}{\partial \rho_i}} &= \frac{1+g_i^b}{1-g_i^a}\frac{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i + \frac{Q}{\rho_i}}{\left(1-\frac{1}{\lambda_i}\right)\mathbf{A}_i - \frac{Q}{\rho_i}} > 1, \\
\frac{\frac{\partial g_i^b}{\partial \lambda_i}}{\frac{\partial g_i^a}{\partial \lambda_i}} &= \frac{1+g_i^b}{1-g_i^a}\frac{1}{1-\rho_i}\frac{\mathbf{A}_i + Q\frac{\lambda_i}{\lambda_i-1}}{\mathbf{A}_i}\frac{\mathbf{A}_i + \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}}{\mathbf{A}_i - \frac{Q}{\rho_i}\frac{\lambda_i}{\lambda_i-1}} > 1,
\end{aligned}
$$

proving this relationship. Using the same steps as before we obtain

$$\begin{aligned}
\frac{\partial r_i^a}{\partial Q} &= g_i^a \frac{\partial f_i}{\partial Q} + \frac{\partial g_i^a}{\partial Q} f_i, \\
\frac{\partial r_i^b}{\partial Q} &= g_i^b \frac{\partial f_i}{\partial Q} + \frac{\partial g_i^b}{\partial Q} f_i,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial f_i}{\partial Q} &= -\frac{\mathbf{E}_i \left(1 + r_i^E\right)}{Q^2} = -\frac{f_i}{Q}, \\
\frac{\partial g_i^a}{\partial Q} &= \frac{\theta_i}{1-\theta_i} \left(1 - g_i^a\right) \frac{\mathbf{D}_i + \mathbf{B}_i - \mathbf{R}_i}{\left(\mathbf{R}_i + Q\right)\left(\mathbf{D}_i + \mathbf{B}_i + Q\right)} > 0, \\
\frac{\partial g_i^b}{\partial Q} &= \frac{\theta_i}{1-\theta_i} \left(1 + g_i^b\right) \frac{1}{\mathbf{R}_i - Q} > 0.
\end{aligned}$$

Inserting into the equations we get

$$\begin{aligned}
\frac{\partial r_i^a}{\partial Q} &= -g_i^a \frac{f_i}{Q} + \frac{\partial g_i^a}{\partial Q} f_i = \left(-\frac{g_i^a}{Q} + \frac{\partial g_i^a}{\partial Q}\right) f_i, \\
\frac{\partial r_i^b}{\partial Q} &= -g_i^b \frac{f_i}{Q} + \frac{\partial g_i^b}{\partial Q} f_i = \left(-\frac{g_i^b}{Q} + \frac{\partial g_i^b}{\partial Q}\right) f_i,
\end{aligned}$$

and it remains to be shown that $\frac{\partial g_i^a}{\partial Q} < \frac{g_i^a}{Q}$ and $\frac{\partial g_i^b}{\partial Q} > \frac{g_i^b}{Q}$.

The second derivatives of g_i^a and g_i^b are given by

$$\begin{aligned}
\frac{\partial^2 g_i^a}{\partial Q^2} &= \frac{\theta_i}{1-\theta_i} \left(-\frac{\theta_i}{1-\theta_i} \left(1 - g_i^a\right) \frac{\left(\mathbf{D}_i + \mathbf{B}_i - \mathbf{R}_i\right)^2}{\left(\mathbf{R}_i + Q\right)^2 \left(\mathbf{D}_i + \mathbf{B}_i + Q\right)^2} \right. \\
&\qquad \left. + \left(1 - g_i^a\right) \frac{\left(\mathbf{R}_i + Q\right)^2 - \left(\mathbf{D}_i + \mathbf{B}_i + Q\right)^2}{\left(\mathbf{R}_i + Q\right)^2 \left(\mathbf{D}_i + \mathbf{B}_i + Q\right)^2}\right) < 0, \\
\frac{\partial^2 g_i^b}{\partial Q^2} &= \frac{\theta_i}{(1-\theta_i)^2} \left(g_i^b + 1\right) \frac{1}{\left(\mathbf{R}_i - Q\right)^2} > 0.
\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\partial g_i^a}{\partial Q} Q &= \int_0^Q \frac{\partial g_i^a}{\partial Q} dq < \int_0^{q_i} \left.\frac{\partial g_i^a}{\partial Q}\right|_{Q=q} dq = g_i^a, \\
\frac{\partial g_i^b}{\partial Q} Q &= \int_0^Q \frac{\partial g_i^b}{\partial Q} dq > \int_0^{q_i} \left.\frac{\partial g_i^b}{\partial Q}\right|_{Q=q} dq = g_i^b,
\end{aligned}$$

completing the proof.

A.6 Lemma 4

Consider the transaction between two banks, denoted 1 and 2. Before transactions, bank i has reservation rates r_i^a and r_i^b, respectively. In order for a transaction to happen, it has to be either $r_1^a \geq r_2^b$ or $r_1^b \leq r_2^a$. Without loss of generality, we assume that $r_1^b \leq r_2^a$, so bank 1 lends to bank 2. The actual agreed interest rate r_{tr} has to satisfy $r_1^b \leq r_{tr} \leq r_2^a$. After this transaction, bank 1 has reservation rates $(r_{1,L}^a, r_{1,L}^b)$ and bank 2 has reservation rates $(r_{2,B}^a, r_{2,B}^b)$. We need only to show $r_{1,L}^a < r_{2,B}^b$, to make sure bank 1 would not borrow from bank 2.

Noting that due to the assumption that banks have identical preferences we have $\theta_1 = \theta_2 = \theta$, after the transaction of size Q the reservation prices are given by

$$\begin{aligned}
r_{1,L}^a - r^f &= \left(1 - \left(\frac{\mathbf{R}_1 - Q}{\mathbf{R}_1}\left(1 + \frac{Q}{\mathbf{D}_1 + \mathbf{B}_1}\right)\right)^{\frac{\theta}{1-\theta}}\right) \\
&\quad \times \left(\frac{\mathbf{E}_1}{Q}\left(1 + r_1^E\right) + r_{tr} - r^f\right), \\
r_{1,L}^b - r^f &= \left(\left(\frac{\mathbf{R}_1 - Q}{\mathbf{R}_1 - 2Q}\right)^{\frac{\theta}{1-\theta}} - 1\right)\left(\frac{\mathbf{E}_1}{Q}\left(1 + r_1^E\right) + r_{tr} - r^f\right), \\
r_{2,B}^a - r^f &= \left(1 - \left(\frac{\mathbf{R}_2 + Q}{\mathbf{R}_2 + 2Q}\left(1 + \frac{Q}{\mathbf{D}_2 + \mathbf{B}_2 + Q}\right)\right)^{\frac{\theta}{1-\theta}}\right) \\
&\quad \times \left(\frac{\mathbf{E}_2}{Q}\left(1 + r_2^E\right) - r_{tr} - r^f\right), \\
r_{2,B}^b - r^f &= \left(\left(\frac{\mathbf{R}_2 + Q}{\mathbf{R}_2}\right)^{\frac{\theta}{1-\theta}} - 1\right)\left(\frac{\mathbf{E}_2}{Q}\left(1 + r_2^E\right) - r_{tr} - r^f\right).
\end{aligned} \tag{13}$$

We denote the transaction price by a linear combination of r_1^b and r_2^a such that with $0 \leq \omega \leq 1$

$$r_{tr} - r^f = \omega(r_1^b - r^f) + (1 - \omega)(r_2^a - r^f),$$

and further define $0 < p \leq 1$ implicitly by

$$r_1^b - r^f = p\left(r_2^a - r^f\right).$$

Combining these two equations we obtain

$$r_{tr} - r^f = \left(\omega + \frac{1 - \omega}{p}\right)\left(r_1^b - r^f\right). \tag{14}$$

We can rewrite equation (13) using this notation as

$$
\begin{aligned}
r^a_{1,L} - r^f &= 1 - \left(\frac{\mathbf{R}_1 - Q}{\mathbf{R}_1} \left(1 + \frac{Q}{\mathbf{D}_1 + \mathbf{B}_1} \right) \right)^{\frac{\theta}{1-\theta}} \Bigg) \\
&\quad \times \left(\frac{1}{\left(\frac{\mathbf{R}_1}{\mathbf{R}_1 - Q} \right)^{\frac{\theta}{1-\theta}} - 1} + \left(\omega + \frac{1-\omega}{p} \right) \right) \left(r^b_1 - r^f \right), \\
r^b_{2,B} - r^f &= \left(\left(\frac{\mathbf{R}_2 + Q}{\mathbf{R}_2} \right)^{\frac{\theta}{1-\theta}} - 1 \right) \\
&\quad \times \left(\frac{1}{1 - \left(\frac{\mathbf{R}_2}{\mathbf{R}_2 + Q} \left(1 + \frac{Q}{\mathbf{D}_2 + \mathbf{B}_2} \right) \right)^{\frac{\theta}{1-\theta}}} - (p\omega + 1 - \omega) \right) \left(r^a_2 - r^f \right)
\end{aligned}
$$

and obtain using (14)

$$
\begin{aligned}
r^a_{1,L} - r_{tr} &= \Bigg\{ \left(1 - \left(\frac{\mathbf{R}_1 - Q}{\mathbf{R}_1} \left(1 + \frac{Q}{\mathbf{D}_1 + \mathbf{B}_1} \right) \right)^{\frac{\theta}{1-\theta}} \right) \\
&\quad \times \left(\frac{1}{\left(\frac{R_1}{\mathbf{R}_1 - Q} \right)^{\frac{\theta}{1-\theta}} - 1} + \left(\omega + \frac{1-\omega}{p} \right) \right) - \omega + \frac{1-\omega}{p} \Bigg\} \left(r^b_1 - r^f \right), \\
r^b_{2,B} - r_{tr} &= \Bigg\{ \left(\left(\frac{\mathbf{R}_2 + Q}{\mathbf{R}_2} \right)^{\frac{\theta}{1-\theta}} - 1 \right) \\
&\quad \times \left(\frac{1}{1 - \left(\frac{\mathbf{R}_2}{\mathbf{R}_2 + Q} \left(1 + \frac{Q}{\mathbf{D}_2 + \mathbf{B}_2} \right) \right)^{\frac{\theta}{1-\theta}}} - (p\omega + 1 - \omega) \right) - (p\omega + 1 - \omega) \Bigg\} \\
&\quad \times \left(r^a_2 - r^f \right).
\end{aligned}
$$

Now we need only to show the above equations to be negative and positive, respectively. Dropping $r^b_1 - r^f > 0$ and $r^a_1 - r^f > 0$ from these equations we obtain the conditions as

$$
\begin{aligned}
&1-\left(\frac{\mathbf{R}_1-Q}{R_1}\left(1+\frac{Q}{\mathbf{D}_1+\mathbf{B}_1}\right)\right)^{\frac{\theta}{1-\theta}}\Bigg) \\
&\times\left(\frac{1}{\left(\frac{\mathbf{R}_1}{\mathbf{R}_1-Q}\right)^{\frac{\theta}{1-\theta}}-1}+\left(\omega+\frac{1-\omega}{p}\right)\right)-\omega+\frac{1-\omega}{p}<0, \\
&\left(\left(\frac{\mathbf{R}_2+Q}{\mathbf{R}_2}\right)^{\frac{\theta}{1-\theta}}-1\right) \\
&\times\left(\frac{1}{1-\left(\frac{\mathbf{R}_2}{\mathbf{R}_2+Q}\left(1+\frac{Q}{\mathbf{D}_2+\mathbf{B}_2}\right)\right)^{\frac{\theta}{1-\theta}}}-p\omega+1-\omega\right)-p\omega+1-\omega>0.
\end{aligned}
$$

After some transformations we can easily see that these relationships hold.

B Appendix: Robustness for multicollineraity

For regressions in table 4, since the values of these independent variables are determined exogenously, dependent variables such as error score, core size or core centrality cannot influence their values. Therefore, there cannot exist an endogeneity issue in these regressions. We also consider the possibility of multicollinearity affecting the R^2 by investigating the correlation matrix given as below. We find the correlation between any two independent variables to be quite weak (no more than 0.25 and in many cases less than 0.1). This is consistent with the setting of our experiments as these independent variables are randomly and independently assigned. The only exception is the correlation between r^f and r^C which is moderate at 0.4854. This is expected since r^C is set as r^f plus an random amount in order to make sure r^C is over r^f within reasonable range. We also tried an alternative regression that does not include r^f as an independent variable in table 7 and find it produces a similar result as the original regression. Specifically, estimates in the alternative regression have the same sign and similar values as the original, and the R^2 is not much affected either. Our analysis suggests the high level of R^2 of the original regression is very likely the result of good fit.

Correlation	$\log(\theta)$	$\log(Q)$	$\log(N)$	$\frac{1}{\lambda}$	β	r^f	r^D	r^C
$\log(\theta)$	1.0000							
$\log(Q)$	0.1116	1.0000						
$\log(N)$	−0.0179	0.0294	1.0000					
$\frac{1}{\lambda}$	−0.1536	−0.0788	0.0186	1.0000				
β	−0.2302	−0.0156	−0.0385	−0.2198	1.0000			
r^f	−0.0406	−0.0141	−0.0188	−0.0127	−0.0146	1.0000		
r^D	0.0186	0.0208	−0.0484	−0.0012	0.0241	−0.0092	1.0000	
r^C	0.1820	−0.0391	−0.0344	0.1369	0.0871	0.4854	0.0008	1.0000

Table 6: Correlation matrix of variables in table 4.

	log(Error score)	log(Core size)	log(Core centrality)
Constant	2.6942	2.1256	−1.1481
$\log(\theta)$	0.7791	0.1076	0.0254
$\log(Q)$	−0.2895	−0.6080	0.3563
$\log(N)$	−0.2936	−0.8504	−0.4929
$\frac{1}{\lambda}$	3.4587	−0.6712	0.8325
β	0.0260	−0.0692	0.0332
r^D	−2.1083	−0.9005	−0.2205
r^C	−5.6842	−0.5293	−0.4993
R^2	0.6634	0.8646	0.8284

Table 7: OLS regression as in table 4 with r^f excluded.

Chapter 4

Essay 3 - Bank Demand for Central Bank Liquidity: The Impact of Bid Shading and Interbank Markets

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Contents

Bank Demand for Central Bank Liquidity: The Impact of Bid Shading and Interbank Markets

Di Xiao[*] Andreas Krause[†]

Abstract

We develop a model in which banks bid for liquidity provided by the central bank in fixed and variable rate auctions, considering liquidity injections and extractions as well as the impact of a subsequent interbank market. We derive the equilibrium demands of banks establishing the prevalence of bid shading and show that fixed and variable rate tenders lead to the same allocation in equilibrium. We also investigate the impact the central bank auction has on the subsequent interbank market and find that while lending in the interbank market is reduced, the interest rates are moving in the desired direction. The impact of monetary policy on banks located in the core and the periphery of the interbank network are different.

Keywords: Central bank operation, multi-unit auction, fixed rate auction, variable rate auction, interbank network, core-periphery network

1 Introduction

In the aftermath of the global financial crisis 2007/8 central banks around the world provided banks with significant amounts of liquidity to counter the freeze in the interbank market, but also in order to maintain low interest rates. There has been considerable effort to assess the impact this liquidity injection had on the lending behaviour of banks and the wider economy, see e. g. Bernanke & Blinder (1992), Sims (1992), Kashyap & Stein (2000), Ehrmann et al. (2001), Rudebusch & Wu (2008), and Carpenter et al. (2014), but what has received very little attention,

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however, is the impact this monetary policy had on the interbank market structure. Understanding this link between monetary policy and interbank markets is paramount also to assess the impact of such monetary policy on systemic risk but also how the reversal of the very loose monetary policy affects banks.

This paper seeks to address the question of the banks' demand for central bank liquidity and how it impacts on the interbank market. We will consider the strategic behaviour of banks when bidding for central bank liquidity, and also include the impact of an interbank market for liquidity on this behaviour. The importance of the interbank market for systemic risk has been explored in Freixas et al. (2000), Allen & Gale (2000), Furfine (2003), Battiston et al. (2012), Georg (2013), and Acemoglu et al. (2015), amongst others. In these contributions it has become clear that the structure of the interbank market affects the level of systemic risk. Most notably Fricke & Lux (2015), Craig & Von Peter (2014), and Langfield et al. (2014) have established that the interbank market exhibits a core-periphery structure, i. e. a small number of banks are highly interconnected and form the core while the vast majority of banks only connect to this core but not other banks in this periphery. This paper will explore how monetary policy affects the structure of the interbank market as well as interest rates in interbank markets.

One important feature of our model is that banks have preference for both liquidity and returns. In other words, we assume just as banks seek to maximize profitability, illiquidity is also undesirable. Consequently, banks in our model face an internal trade off between these two objectives when maximizing utility. This is unlike other comparable studies where banks only optimize returns but are essentially indifferent about their liquidity level. It also has important implications for the modelling of the demand for central bank funds and the resulting equilibrium.

In our paper we consider monetary policy in form of the auction of short term central bank funds prior to the commencement of the interbank market. The central bank may choose to implement either liquidity injection or liquidity extraction via auctions to conduct their monetary policy, the extent of which is determined exogenously. Banks face an idiosyncratic liquidity shock leading to different valuations of liquidity and subsequently different bid schedules for central bank funds. Any demand for liquidity that is not met by the central bank through their auction facility can in a second step be offset in the interbank market. Thus our model combines in a new way the demand for liquidity in central bank operations with that of the interbank market and shows the interactions between these two facilities.

We continue as follows: the following section provides a brief overview of the relevant literature, while section 3 details the model for the demand of central bank funds and section 4 assesses the results of our model in the interbank market. Finally section 5 concludes our findings. All proofs are provided in the appendix.

2 Literature Review

The focus of our model is on the short-term funding provided by the central bank to commercial banks. This a wide range of mechanisms are employed around the world, most notably open-market purchases by the US Federal Reserve, the mechanism that provides the basis of our model is most closely resembling that of the European Central Bank (ECB). The ECB uses a system of repurchase agreements with a maturity of one week. To allocate the funding in these repurchase agreements the ECB conducts them either in fixed rate or variable rate tenders, where in fixed rate tenders the ECB specifies the interest applicable and banks bid for the volume they want to obtain at these conditions with the bank allocating the amounts subject to a global limit on the provision of liquidity. In contrast to that, in variable rate tenders the ECB specifies the total quantity of liquidity to be supplied and banks submit a bid schedule specifying the amount and interest rate they are requesting, subject to a minimum interest rate and a maximum number of ten different bids. In all cases banks have to provide collateral and be financially stable.

Until June 2000 the ECB used only fixed rate tenders and subsequently changed to variable rate tenders until October 2008. After this date in response to the financial crisis the ECB reversed to fixed rate tenders and allowed banks to bid for and be allocated unlimited amounts, provided it has sufficient collateral and is financially stable. The ECB also operates a more long term provision of liquidity to banks for the duration of three months, also by variable or fixed rate tenders. For more details on the operation of the ECB's operation of monetary policy see European Central Bank (2011).

2.1 Central bank operation using auctions

Despite the importance of auctions to provide liquidity to banks, the literature investigating these mechanisms either theoretically or empirically are relatively limited. Nautz & Oechssler (2003) introduce a model of banks strategically bidding in a fixed rate tender where banks minimise the deviations between the liquidity acquired in the auction and the liquidity they desire. Although they find the resulting game does not have an equilibrium, they also demonstrate that banks increasingly exaggerate their demand with an adaptive bidding rule. Ayuso & Repullo (2003) extend this model by including an expectation of interbank market rates, assuming the interbank market to be efficient. Furthermore they assume that the central bank minimises a loss function of the difference between the interbank rate and a target policy rate, and find that if the loss function punishes more heavily when interbank rate move below rather than above the target (which has similar effect as rationing), fixed rate tenders have a unique equilibrium with high overbidding.

Nyborg & Strebulaev (2003) take a different approach that allows for a brief squeeze in the interbank market commencing after the auction. They also differ from the above model as they assume banks to maximize interest earnings. They find that pre-auction positions can affect a bidder's behaviour in equilibrium. Specifically, bidders with short positions tend to bid more aggressively due to the concern of experiencing a loss of access to sufficient liquidity in the interbank market. Ewerhart et al. (2010) take an alternative approach, assuming collateral to be heterogeneous and central bank funds supply to be uncertain. Banks with a goal to maximize interest earnings can either get the liquidity in the auction or alternatively in the interbank market at a cost of putting up more expensive collateral. In equilibrium, their model also predicts bid shading, i. e. the submission of bids that do not reflect their true preferences. While this model is close to ours as it uses a private value for liquidity. However, our model differs substantially in that they assume a bank's valuation for funds is based on the cost of using collateral while our model will be driven by the desire of a high profitability and high liquidity.

Empirically a number of properties have been found that a model of such auctions should capture: overbidding, bid shading and flat bids. *Overbidding*, i. e. requesting more funds than required in anticipation of rationing, is observed empirically in fixed rate tenders with (possible) rationing, as shown in Ayuso & Repullo (2003) and Nautz & Oechssler (2003). They also found the switch by the ECB from fixed rate to variable tenders mitigated overbidding without losing much control over interbank rates.

Bid shading is when banks submit bids for liquidity that do not fully reflect their true preferences. They will submit bids that show a lower willingness to pay for a given quantity than their preferences would imply in order to improve their utility from this auction. Empirically bid shading is usually measured by the differences between the auction rate and subsequent interbank market rate. The evidence on bid shading is mixed and depends on the sample period and the index of interbank market rates used. For instance, Ayuso & Repullo (2003) use the 1-week Euribor and Eonia on the day of settlement of the central bank operation. For both fixed and variable rate tenders the interbank rate is 3 (Eonia) or 4 (Euribor) basis points above the average tender rate and the latter is significant. Bindseil et al. (2009) use swap rates 15 minutes before auctions and find them 3.33bp higher than the weighted average bid and 1.64 bp higher than the weighted average winning bid. In contrast, Nautz & Oechssler (2003) use Eonia on the day of announcement of the central bank operation and find it to be very close to the marginal rate of the ECB's variable rate tender. Therefore they argue the difference is not empirically relevant and the small difference could be due to differences in collateral requirements. Cassola et al. (2013) cover the height of the financial crisis in 2007 and find that the spread between bank bids and Eonia is 4bp on average. Moreover, they find that after the financial crisis this spread increases to 10 basis points. They argue that the central bank operation resembles an auction of a common value good where bidders have private information. The interbank market rate is viewed as this "common value" of liquidity. Bidders strategically shade their bids to avoid the "winner's curse" in this auction and thus bid shading would increase with uncertainty about the common value. However, Bindseil et al. (2009) find no support for this in empirical evidence, suggesting this framework may not fit central bank operations.

In the ECB's variable rate tender a bank can submit a bid schedule of up to 10 bids, but banks seldom utilize all of them. The average number of bids is 2 to 3 and the bid schedule is quite *flat* as reported in Bindseil et al. (2009) and Cassola et al. (2013). The average winning tender rate is only 1.7bp above the marginal tender rate according to Ayuso & Repullo (2003).

There have also been a small number of investigations on the relationship between central bank auctions for liquidity and the interbank market. Brunetti et al. (2010) and Linzert & Schmidt (2011) have shown that in the Euro zone area, prior to the crisis period of 2007/8, central bank interventions are usually reducing interbank spreads. For the US, where the Federal Reserve used a term auction facility for maturities of one to three months in response to the financial crisis 2007/8, McAndrews et al. (2017) and Wu (2008) find that liquidity injections reduce the interbank spread, even if excluding the credit risk associated with interbank markets. Taylor & Williams (2009) find the opposite effect, but McAndrews et al. (2017) suggest their model is incorrectly specified.

2.2 Multi-unit auction theory

The theoretical framework for the demand for central bank funds through auctions is auction theory. As banks can request different amounts of liquidity, the auctions are for multiple units, hence theories of auctions of a perfectly divisible good are the appropriate theoretical framework. Such a multi-unit auction problem was first studied by Wilson (1979), where a known number of symmetric bidders bid for shares of an item of common value. He first considers uniform pricing and discusses cases where the item value is certain and where it is not. He also discuss cases where bidders have proprietary information and where they have not. In all these cases, he finds that there is some equilibrium where the sale price of the item is a lot lower than if the auction is a single-unit auction. In other words, in such an equilibrium a bidder's strategy is not to reveal their true value of the item (bid shading). On the contrary, bidders can "collusively"' shade their true demand and be better off. It is worth mentioning, in a single-unit auction of a good of common value there is also an incentive for bidders to shade their bids when bidders have a noisy signals of the good's value. This is because the bidder who wins the auction must have a signal that is the highest value and can still win by paying a bit less (the "winner's curse") and thus it is not optimal to bid according to one's signal.

The results from Wilson (1979) have been generalized in Back & Zender (1993) and alternative information settings been applied in Back & Zender (2001). A framework of private value goods instead of common value goods has also been investigated. For instance, the split award procurement auction is studied in both a complete information setting in Anton & Yao (1989) and an incomplete information setting by Anton & Yao (1992). There are also a number of studies that consider endogenous supply and find it helps to reduce bid shading, e. g. Klemperer & Meyer (1989) and Back & Zender (2001). A major topic in this area is the comparison between uniform and discriminatory pricing mechanism (comparable to fixed and variable rate tenders in our model) in terms of which is optimal, e.g. Tenorio (1997), Back & Zender (2001), or Ausubel et al. (2014). Studies on treasury auctions in

Binmore & Swierzbinski (2000), Abbink et al. (2006), Goldreich (2007), Hortaçsu & McAdams (2010), and Kang & Puller (2008) as well as electricity markets by Federico & Rahman (2003) and Fabra et al. (2006) also continue the debate from auction theory. However, the general finding is inconclusive as the results depend on the detailed assumptions about bidders and the auction mechanism itself as pointed out in Ausubel et al. (2014).

3 A model of central bank borrowing

We consider a banking system with $N > 2$ banks where each bank i seeks to maximize its utility, which consists of two elements. Firstly banks seek to maximize their profitability measured by the return on equity. Using the stylized balance sheet of a bank from table 1, we can define the return on equity as

$$r_i^E = \frac{(\mathbf{R}_i + Q_i)\, r^f + \mathbf{L}_i r_i^L - Q_i r_i^{CB} + \mathbf{C}_i r_i^C - \mathbf{D}_i r_i^D - \mathbf{B}_i r_i^B}{\mathbf{E}_i}, \tag{1}$$

where r^f is the risk free rate, r_i^L is the weighted average rate on interbank lending, r_i^{CB} is the average rate on central bank funds, r_i^C is the average rate on external asset, r_i^D is the average rate paid on external deposit, r_i^B is the weighted average rate on interbank borrowing, and Q_i is the amount of central bank funds with positive numbers indicating a loan from the central bank a negative number a deposit at the central bank.

Secondly, banks will seek to hold large cash reserves as that allows them to withstand any large withdrawals of deposits without having to resort to costly asset liquidation or declare themselves illiquid. We define the cash reserve ratio as

$$\rho_i = \frac{\mathbf{R}_i}{\mathbf{D}_i + \mathbf{B}_i + \max\{0, Q_i\}}. \tag{2}$$

Obviously large cash reserves reduce profitability as the interest paid on these will be smaller than on other investment opportunities. To balance these two aspects we use the following utility function:

$$U_i\left(\rho_i, r_i^E\right) = \gamma_i \rho_i^{\theta_i} \left(1 + r_i^E\right)^{1-\theta_i}, \tag{3}$$

Bank i			
Cash reserves	$\mathbf{R}_i$	Deposits	$\mathbf{D}_i$
Interbank lending	$\mathbf{L}_i$	Interbank borrowing	$\mathbf{B}_i$
Central bank deposit	$\max\{0, -Q_i\}$	Central bank funds	$\max\{0, Q_i\}$
Loans	$\mathbf{C}_i$	Equity	$\mathbf{E}_i$
Total assets	$\mathbf{A}_i$		$\mathbf{A}_i$

Table 1: Bank i's stylized balance sheet with central bank operation

where $0 < \theta_i < 1$ denotes the relative importance of concerns of liquidity relative to profitability, $\gamma_i > 0$ is simply a scaling factor. These banks face non-optimal liquidity holdings, e.g. due to a liquidity shock. We assume that the size and sign of this liquidity shock is common knowledge for all banks. We acknowledge this assumption has some limitations, yet it allows us to derive analytical solutions that are otherwise impossible to obtain. We require that a bank has complete information about how liquidity is distributed in the banking system without matching liquidity positions to each bank's identity. In real markets, when the number of banks in a healthy banking system is large enough, this distribution should be stable. Additionally, experienced banks should have a good estimate of this distribution based on past information (past bids, for instance, are public information). However, this assumption may not apply to a banking system where the overall liquidity condition varies greatly in short time. In such a scenario, past information is not so useful to know the how liquidity is distributed among banks and the assumption of complete information may not be valid.

In addition to these commercial banks, we introduce a central bank into the banking system. The purpose of this central bank is to conduct its monetary policy by increasing or reducing the amount of liquidity in the banking system. How the central bank makes this decision is beyond our scope and we take this decision as exogenously given. Using this approach allows us to focus on how banks react to the decision of the central bank and how it affects interbank markets.

In order to assess the commercial banks' behaviour we will investigate the injection and extraction of liquidity by central banks through fixed rate and variable rate tenders and will also compare models in which banks neglect the existence of the interbank market that opens after the central bank intervention and a model where they consider the opportunities in this market.

3.1 Fixed rate tenders

In a fixed rate tender the central bank determines an interest rate at which it will lend (borrow) to (from) commercial banks and anyone willing to pay (receive) this interest rate will be able to do so. Banks do not know the interest rate the central bank applies and will thus submit a bid schedule for each possible interest rate, i.e. specify the quantity they demand.

If we use $Q_i > 0$ to denote borrowing from the central bank and $Q_i < 0$ for depositing additional funds, we know from Xiao & Krause (2017) that for an amount of Q_i the reservation price of banks is given by the following expression:

$$r_i^a\left(Q_i\right) = r^f + \left. 1 - \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q_i}\left(1 + \frac{Q_i}{\mathbf{D}_i + \mathbf{B}_i}\right)\right)^{\frac{\theta_i}{1-\theta_i}}\right) \frac{\mathbf{E}_i\left(1 + r_i^E\right)}{Q_i} \tag{4}$$

for $Q_i > 0$ and

$$r_i^b\left(Q_i\right) = r^f + \left(\left(1 - \frac{\mathbf{R}_i}{\mathbf{R}_i + Q_i}\right)^{\frac{\theta_i}{1-\theta_i}}\right) \frac{\mathbf{E}_i\left(1 + r_i^E\right)}{Q_i} \tag{5}$$

for $Q_i < 0$. It is shown in Xiao & Krause (2017) that $r^f < r_i^a < r_i^b$, i. e. the bid-ask spread is always positive. We can easily see this still holds even when Q_i approaches 0 as shown in the following lemma:

Lemma 1. $\lim_{Q_i \to 0} r_i^b\left(Q_i\right) - r_i^a\left(Q_i\right) > 0$.

Obviously banks would not bid at their reservation prices, as this would not allow the banks to increase their utility level. Hence we would require banks to maximize their utility by submitting their bids strategically.

$$\begin{aligned}
\max_{Q_i} \quad & U_i\left(\rho_i, r_i^E\right) \\
\text{s.t.} \quad & \left(1 + r_{i,0}^E\right)\mathbf{E}_i + Q_i\left(r^f - r\right) \geq 0 \\
& Q_i \leq \bar{Q}_i \text{ if } Q_i \geq 0 \\
& -Q_i \leq \mathbf{R}_i \text{ if } Q_i \leq 0
\end{aligned} \tag{6}$$

The first constraint ensures that the bank remains solvent, i.e. any losses it might make does not exceed its equity, while the second constraint ensures that any borrowing does not exceed any limit set by the central bank on borrowing, $\bar{Q}_i$, e.g. resulting from absolute limits, limits on leverage, or collateral requirements. The final constraint ensures that the bank does not seek to deposit more funds within the central bank than it has cash reserves available.

Conducting this optimization we obtain the demand for central bank money as detailed in the following proposition:

Proposition 1. *Let* $\psi_i = \frac{1}{2}\left(\frac{1}{1-\theta_i}\left(\mathbf{D}_i + \mathbf{B}_i\right) + \frac{1-2\theta_i}{1-\theta_i}\mathbf{R}_i\right)$ *and* $\varphi = \psi_i^2 - \left(\mathbf{D}_i + \mathbf{B}_i\right)\mathbf{R}_i \frac{r_f^{CB} - r_i^a(0)}{r_f^{CB} - r^f}$. *The equilibrium bid schedule is then given by*

$$Q_i^f\left(r_f^{CB}\right) = \begin{cases} \min\left\{\overline{Q}_i, -\psi_i + \varphi^{\frac{1}{2}}\right\} & \text{if } r^f < r_f^{CB} < r_i^a(0) \\ 0 & \text{if } r_i^a(0) \leq r_f^{CB} \leq r_i^b(0) \\ \theta_i \frac{\left(1 + r_i^E\right)\mathbf{E}_i}{r_f^{CB} - r^f} - \left(1 - \theta_i\right)\mathbf{R}_i & \text{if } r_i^b(0) < r_f^{CB} \end{cases}$$

with $\frac{\partial Q_i^f\left(r_f^{CB}\right)}{\partial r_f^{CB}} \leq 0$.

Here note that Q_i^f contains both demand for central bank funds ($Q_i > 0$) as well as deposits with the central bank ($Q_i < 0$). If the central bank is conducting liquidity injection, i. e. $Q^{CB} > 0$, only demand for liquidity is accepted as a bid, hence bank i should submit $\max\left\{0, Q_i^f\right\}$. If the central bank is conducting liquidity extraction, i. e. $Q^{CB} < 0$, banks can only deposit funds and should submit $\min\left\{0, Q_i^f\right\}$.

The equilibrium is then trivially determined at the interest rate such that $\sum_{i=1}^N Q_i^f\left(r_f^{CB}\right) = Q^{CB}$; due to the monotonicity of the bid schedule this equilibrium will be unique.

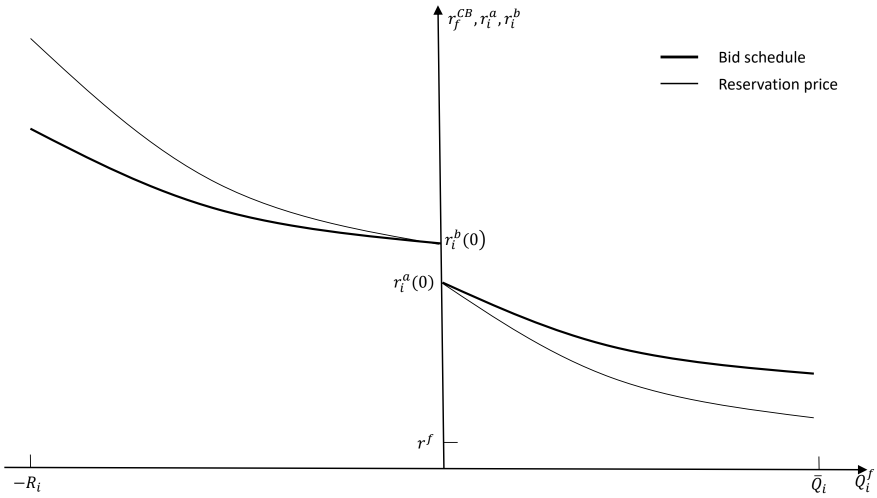

Figure 1: Reservation and equilibrium bid schedules in fixed rate tenders

By inverting the demand function for central bank funds we can easily see that the demand for a given interest rate is lower than the reservation price. Equivalently, the interest rate that can be charged by the central bank for banks borrowing (lending) must be smaller (higher) if they want to increase (decrease) the liquidity by a given amount. This result as detailed in lemma 2 below.

Lemma 2. *The inverse bid schedule in equilibrium is given by*

$$r_f^{CB}\left(Q_i^f\right) = \begin{cases} r^f + \mathbf{E}_i\left(1+r_i^E\right)\left(Q_i^f + \left(\frac{\theta_i}{1-\theta_i}\frac{\mathbf{D}_i+\mathbf{B}_i-\mathbf{R}_i}{\left(\mathbf{R}_i+Q_i^f\right)\left(\mathbf{D}_i+\mathbf{B}_i+Q_i^f\right)}\right)^{-1}\right)^{-1} & \text{if} \quad 0 \leq Q_i^f \leq \overline{Q}_i \\ r^f + \frac{\theta_i}{1-\theta_i}\frac{\mathbf{E}_i\left(1+r_i^E\right)}{\mathbf{R}_i+Q_i^f} & \text{if} \quad Q_i^f < 0 \end{cases}. \tag{7}$$

Figure 1 illustrates this so-called "bid shading", where banks submit their bid schedules strategically in order to maximize their utility and do not submit their reservation prices. As we do not consider the objective function of the central bank in our model, we cannot analyze the welfare implication of this behaviour as any losses suffered by the central bank cannot be quantified.

We also note from figure 1 that the reservation prices, as well as the optimal prices, exhibit a jump at $Q_i = 0$. This jump is the equivalent of the bid-ask spread as explained in lemma 1. The reason for this discontinuity is that the liquidity ratio ρ_i has different properties either side of $Q_i^f = 0$. If bank i deposits money with the central bank ($Q_i^f < 0$) only cash changes affect ρ_i. On the other hand if bank i is borrowing from the central bank, both cash and the total assets change, both affecting ρ_i. Because of this the derivative of ρ_i with respect to Q_i^f is not continuous at $Q_i = 0$ which results in a jump observed in both bank i's reservation price and optimal bid schedule. This result is summarized in the following lemma:

Lemma 3. *The equilibrium rates have the following properties:*

1. $\lim_{Q_i^f \to 0^-} r_f^{CB}\left(Q_i^f\right) = \lim_{Q_i^f \to 0} r_i^b\left(Q_i^f\right)$,
2. $\lim_{Q_i^f \to 0^+} r_f^{CB}\left(Q_i^f\right) = \lim_{Q_i^f \to 0} r_i^a\left(Q_i^f\right)$,
3. $\lim_{Q_i^f \to 0^-} r_f^{CB}\left(Q_i^f\right) > \lim_{Q_i^f \to 0^+} r_f^{CB}\left(Q_i^f\right)$,
4. $\forall Q_i^f \leq 0 : r_i^b\left(Q_i^f\right) \leq r_f^{CB}\left(Q_i^f\right)$,
5. $\forall Q_i^f \geq 0 : r_i^a\left(Q_i^f\right) \geq r_f^{CB}\left(Q_i^f\right)$.

While these considerations are based on the inverse bid schedule, it is easy to revert back to the actual bid schedule. We thus have established that in fixed rate tenders bid shading exists in that banks submit bids for lower quantities at a given tender rate in the case of depositing with the central banks as well as borrowing from the central bank. We could also establish that moving from depositing with the central bank to borrowing from it, will involve a discontinuity on the bid schedule in that banks are requiring a significantly higher interest rate to deposit a small amount with the central bank than to borrow a small amount.

We will now in the coming section conduct the same analysis for variable rate tenders and will then be in a position to compare the results of these two auction forms.

3.2 Variable rate tenders

In variable rate tenders the central bank exogenously fixes the total amount of liquidity extracted or injected at Q^{CB}. The interest rate is then adjusted such that only those banks can participate that have submitted the highest (lowest) demand schedules for borrowing (depositing). The interest rate is set such that $\sum_{i=1}^{N} Q_i \leq Q^{CB}$. Each bank pays the interest rate at which it has submitted its bids, i.e. pricing is discriminatory and banks will not pay the same price, but according to their bid schedule. Therefore the reservation price of a bank will be the marginal value of any amount obtained. The following lemma determines these marginal prices:

Lemma 4. *In variable rate tenders the marginal prices are given by*

$$\widetilde{r}_i^a(Q_i) = r^f + \left(1 + r_i^E\right) \mathbf{E}_i \frac{\theta_i}{1-\theta_i} \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q_i} \frac{\mathbf{D}_i + \mathbf{B}_i + Q_i}{\mathbf{D}_i + \mathbf{B}_i} \right)^{\frac{\theta_i}{1-\theta_i}} \frac{(\mathbf{D}_i + \mathbf{B}_i - \mathbf{R}_i)}{(\mathbf{R}_i + Q_i)(\mathbf{D}_i + \mathbf{B}_i + Q_i)}$$

for $Q_i > 0$ and

$$\widetilde{r}_i^b(Q_i) = r^f + \left(1 + r_i^E\right) \mathbf{E}_i \frac{\theta_i}{1-\theta_i} \left(\frac{\mathbf{R}_i}{\mathbf{R}_i + Q_i} \right)^{\frac{\theta_i}{1-\theta_i}} \frac{1}{\mathbf{R}_i + Q_i}$$

for $Q_i < 0$.

As before for fixed rate tenders, in order to maximize utility, banks will not submit their marginal prices, but act strategically. Proposition 2 below shows the characterization of one such equilibrium, where we assume that banks know each other's liquidity positions.

Proposition 2. *Let* $\widetilde{r} \in \left\{ r \geq r^f \middle| \sum_{i=1}^{N} \max\left(0, Q_i^f(r)\right) = Q^{CB} \right\}$, $\widetilde{\widetilde{r}} \in \left\{ r \geq r^f \middle| \sum_{i=1}^{N} \min\left(0, Q_i^f(r)\right) = Q^{CB} \right\}$, $\widetilde{Q}_i(r) \in \left\{ Q_i \leq Q_i^f(\widetilde{r}) \middle| U_i\left(\rho_i(Q_i), r_i^E(r, Q_i)\right) = U_i\left(\rho_i\left(Q_i^f(\widetilde{r})\right), r_i^E\left(\widetilde{r}, Q_i^f(\widetilde{r})\right)\right), r > r^f \right\}$, *and* $\widetilde{\widetilde{Q}}_i(r) \in \left\{ Q_i \geq Q_i^f(\widetilde{r}) \middle| U_i\left(\rho_i(Q_i), r_i^E(r, Q_i)\right) = U_i\left(\rho_i\left(Q_i^f(\widetilde{r})\right), r_i^E\left(\widetilde{r}, Q_i^f(\widetilde{r})\right)\right), r > r^f \right\}$.

One Nash equilibrium bid schedule is then given by

- *If $0 < Q^{CB} < (N-1)\bar{Q}_i$ the demand schedule is given by*

$$Q_i^v(r) = \begin{cases} \min\left\{\bar{Q}_i, Q_i^f(\widetilde{r}) + \max_{j=1,\ldots,N}\left(Q_j^f(\widetilde{r}) - \widetilde{Q}_j(r)\right)\right\} & if \quad r^f \leq r < \widetilde{r} \\ Q_i^f(\widetilde{r}) & if \quad r = \widetilde{r} \\ 0 & if \quad r > \widetilde{r} \text{ or } r_i^a(0) \leq \widetilde{r} \end{cases}.$$

- *If* $-\min_j\left(\sum_{i\neq j}^{N}\mathbf{R}_i\right) < Q^{CB} < 0$ *the demand schedule is given by*

$$Q_i^v(r)=\begin{cases}0 & if \quad r^f \leq r < \widetilde{\widetilde{r}} \text{ or } r_i^b(0)\geq \widetilde{\widetilde{r}}\\ Q_i^f\left(\widetilde{\widetilde{r}}\right) & if \quad r=\widetilde{\widetilde{r}}\\ Q_i^f\left(\widetilde{\widetilde{r}}\right)-\max_{j=1,\ldots,N}\left(\widetilde{\widetilde{Q}}_j(r)-Q_j^f\left(\widetilde{\widetilde{r}}\right)\right) & if \quad r>\widetilde{\widetilde{r}}\end{cases}$$

The equilibrium is then trivially determined such that $\sum_{i=1}^{N} Q_i^v(r) = Q^{CB}$. Due to the full information banks have of each other's liquidity position they can fully anticipate the respective demands and submit bids that ensure this equilibrium to be reached.

This demand schedule is not easily interpreted and comparable to the result obtained in the fixed rate tender. Hence we illustrate the equilibrium in figure 2. We see that the bids submitted by the banks are entirely flat at $\widetilde{r}$ and $\widetilde{\widetilde{r}}$, respectively, until the quantity bid reaches Q_i^f. For larger quantities beyond this threshold the rate acceptable would be lower for liquidity injections and higher for liquidity extractions, the exact shape depending on the liquidity shocks and preferences of the banks. This area of the demand schedule has no unique solution for the same allocations and interest rates. Proposition 2 provides one such bid schedule explicitly.

This inverse bid schedule is given more formally in the following lemma.

Lemma 5. *The inverse bid schedule in variable rate tenders is given by*

$$r_v^{CB}(Q_i)=\begin{cases}R^{-1}(Q_i) & if \quad Q_i^f\left(\widetilde{\widetilde{r}}\right)\geq Q_i\\ \widetilde{\widetilde{r}} & if \quad 0>Q_i>Q_i^f\left(\widetilde{\widetilde{r}}\right)\\ \widetilde{r} & if \quad 0<Q_i<Q_i^f\left(\widetilde{r}\right)\\ R^{-1}(Q_i) & if \quad Q_i^f\left(\widetilde{r}\right)\leq Q_i\leq \bar{Q}_i\end{cases},$$

where $R^{-1}(Q_i)$ denotes the inverse function of $Q_i^v(r)$ as defines in proposition 2.

As in the case of fixed rate tenders we observe bid shading by banks, which can easily be verified by comparing the equilibrium in proposition 3 below with the marginal prices in lemma 4.

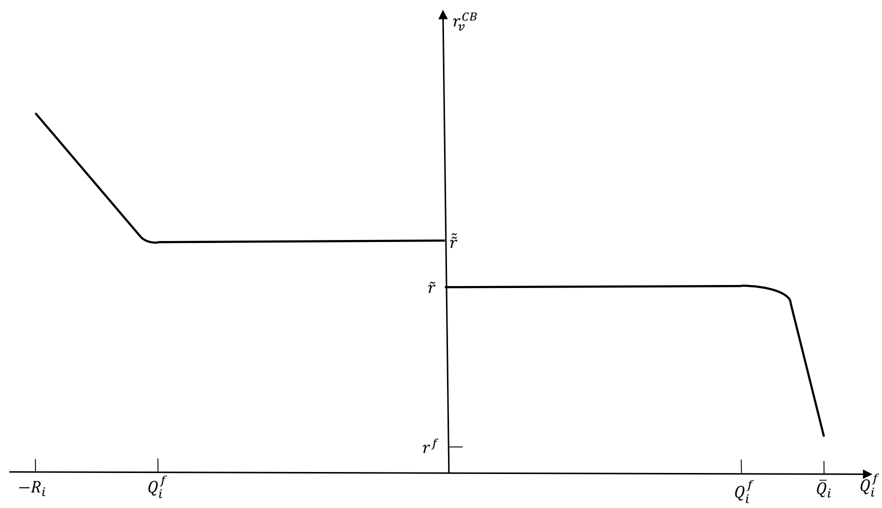

Figure 2: Equilibrium bid schedules in variable rate tenders

Even though, the bid schedules in the two considered tender mechanisms are very different, we can show that the allocation the central bank achieves can be identical in both cases, i.e. each bank obtains the same amount and the same interest rate.

Proposition 3. *An equilibrium exists with* $Q_i^f = Q_i^v$ *and* $r_f^{CB} = r_v^{CB}$.

From a central bank perspective, the two tender formats generate the same revenue. Similar results have been found in single unit auctions with risk neutral participants and a private value framework such as in Vickrey (1961), Holt Jr (1980), or Harris & Raviv (1981). For multi-unit auctions, as in our case, such a result is not generally valid. Which auction mechanism gives the higher revenue can be ambiguous and is also quite sensitive to assumptions about the auction as shown in Ausubel et al. (2014). In our model using the assumption that information of other banks' reservation prices are known leads to not only a tractable equilibrium but also the revenue equivalence of the two auction mechanisms. Given this equivalence between the two tender formats, we will for the remainder of this paper restrict our analysis to the variable rate tender without loss of generality.

Thus far we implicitly assumed that banks ignore the fact that after the bidding for central bank funds an interbank market opens that allows banks to adjust their liquidity holdings. In the following section we will now relax this assumption and allow banks to anticipate this market fully.

3.3 Banks anticipating the interbank market

Until now we have assumed that banks ignore the existence of an interbank market after the central bank intervention in their considerations. If, however, they anticipate that such an interbank market exists, banks would form expectations about the future interbank rate and include this in their utility maximization, thus affecting their bidding for central bank money. In interbank markets the total demand and supply of funds must balance given the bilateral nature of these transactions. Banks, however, have to consider the impact of the central bank contribution Q^{CB} such that they would anticipate a rate of

$$\hat{r}^{IB} \in \left\{ r \left| \sum_{i=1}^{N} \hat{Q}_i = Q^{CB} \right. \right\} \tag{8}$$

where $\hat{Q}_i = Q_i + \hat{Q}_i^{IB}$ is the total amount of liquidity bank i gets from the central bank and it anticipates to get from interbank market. Having anticipated this rate, banks will now engage in variable rate tender bidding considering this rate. Banks are not willing to bid at a rate higher (lower) to borrow from (deposit with) the central bank as waiting for the interbank market would be more profitable. Similarly, banks would not bid at a lower (higher) rate as the additional profits would entice other banks to submit a marginally higher bid, such that competition would ensure the rate submitted to converge towards $\hat{r}^{IB}$.

Thus banks' bid schedules would be flat at $\hat{r}^{IB}$, the amount being such that any rationing in the allocation is fully anticipated. We summarize this outcome in the following proposition.

Proposition 4. *Let* $\widehat{r}^{IB} \in \left\{ r \middle| \sum_i^N Q_i^f = Q^{CB} \right\}$ *and* $\lambda = \frac{Q^{CB}}{\sum_{i=1}^N Q_i^I}$, *then the equilibrium demand for central bank funds is with banks anticipating the is given by*

$\left(\widehat{r}^{IB}, \lambda Q_1^I\left(\widehat{r}^{IB}\right), \ldots, \lambda Q_1^I\left(\widehat{r}^{IB}\right)\right)$, *where*

$$Q_i^I = \begin{cases} \min\left\{\bar{Q}_i, \max\left\{0, Q_i^f\left(\widehat{r}^{IB}\right)\right\}\right\} & if \qquad 0 < Q^{CB} < (N-1)\bar{Q}_i \\ \min\left\{0, Q_i^f\left(\widehat{r}^{IB}\right)\right\} & if \quad -\min_j\left(\sum_{i\neq j}^N \mathbf{R}_i\right) < Q^{CB} < 0 \end{cases}$$

Having established the properties of the equilibrium if banks anticipate the interbank market, we can now continue in the following section to conduct computer experiments of the interbank market itself and see how the existence of the central bank affects this market.

4 The interbank market

We have thus far only assessed the bidding behaviour of banks for central bank funds. In this section we will introduce the mechanism of the subsequent interbank market and then evaluate the impact of the central bank on the properties of this market. The interbank market is set up identically to Xiao & Krause (2017) and we thus briefly characterize its main characteristics to aid the understanding of the institutional setting but refer to their contribution for a detailed characterisation.

We assume that interbank loans are negotiated bilaterally between banks entering the market sequentially in a random order. A bank entering the market will approach all banks in turn in a random order and decide whether to borrow, lend or do nothing at the interest rate that is being quoted. After each transaction the banks involved update the prices at which they are willing to conduct transactions; all transactions are of a fixed size Q^{IB}. After this all banks having been involved in a transaction may then offset the acquired position in further transactions. This process continues until all banks have been approached and involved in a transaction when it is possible. After this the next bank enters the market and the process restarts until no further transactions are possible. The interest rates quoted for a transaction are assumed to be the reservation prices as shown in equations (4) and (5) and a transaction will be possible if the reservation prices of bank i lending and bank j borrowing are such that $r_j^b \leq r_i^a$.

In Xiao & Krause (2017) it is shown that the network of interbank lending normally exhibits a core-periphery structure, i. e. a small number of banks are highly connected with each other (the core) and all other banks (the periphery) connect to these core banks while having very few connections with each other. We will investigate whether this structure is maintained in the presence of a central bank, and will also evaluate other characteristics of the interbank lending network, like the size of the core, the density, but also properties of the interest rates between banks, how much they are lending and borrowing in the interbank market, or the return on equity achieved.

The only difference to the model in Xiao & Krause (2017) is the introduction of a central bank. As in this paper, we assume that banks face an idiosyncratic liquidity shock, using a uniform distribution $U\left(\underline{\rho}, \overline{\rho}\right)$, which they then seek to offset via the central bank and the interbank market. In contrast to Xiao & Krause (2017) this liquidity shock does not have to be balanced on aggregate but will have a positive sign on average. The interbank market cannot be analyzed analytically, hence we use computer experiments in our assessment. We run 8000 such experiments with a wide range of parameter constellations chosen within ranges as detailed in table 2. In order to focus on the effect monetary policy has on the interbank market, we assume that all banks are homogeneous, e. g. have the same size or leverage; they will only differ ex-ante in the idiosyncratic liquidity shock they receive. We assess the impact of the central bank in the cases of liquidity injection and liquidity extraction, both anticipating the existence of a interbank market and not, as well as assessing the interbank market only, that is without the presence of a central bank for comparison purposes.

Looking at the characteristics of the interbank market in table 3 we can see that while

[1]Please note that we include a fixed interest rate r^{CB} rather than the amount Q^{CB} into our computer experiments. Given the demand schedules there is a clear relationship between these two variables and they can be set interchangeably. It was computationally more convenient to exogenously fix the interest rate over the quantity.

$U(a;b)$ denotes a uniform distribution with a lower limit of a and upper limit of b.

Parameter	Symbol	Distribution
Assets	$\mathbf{A}$	fixed at 100
Preferences	θ	$U(0;0.1)$
Leverage	$\frac{1}{\lambda}$	$U(0.01;0.51)$
Number of banks	N	$U(10;1000)$
Risk free rate	r^f	$U(0.025;0.125)$
Deposit rate	r^D	$U(0;r^f)$
Loan rate	r^C	$U(r^f;r^f+0.2)$
Interbank loan size	Q^{IB}	$U(0;2)$
Distribution of liquidity	$\underline{\rho}$	$U(0,0.05)$
	$\overline{\rho}$	$U(\underline{\rho},\underline{\rho}+0.25)$
Collateral constraint	$\overline{Q}$	$U(0.1;0.8)$
Central bank rate[1]	r^{CB}	$U\left(r^f;\frac{1}{N}\sum_{i=1}^N r_i^a(0)\right)$

Table 2: Parameter selection for simulations

the main properties still remain valid in the presence of a central bank conducting its monetary policy, there are some distinct properties that deserve closer attention. Firstly we notice that the injection of liquidity by the central bank reduces the interbank rate while the extraction increases it. This validates the empirical observation that central bank operations affect interbank lending rates. We also note that this effect is stronger for liquidity injections than liquidity extractions. While we observe this effect across lending between all groups of banks, core and periphery, we have the strongest effect on banks in the core lending to those in the periphery. Finally, in the case of liquidity injection the differences in interest rates between core and periphery banks overall reduce, thus the advantages core banks have over periphery banks in terms of profitability from engaging in the interbank market, will also be smaller.

The amount of interbank lending reduces in the presence of a central bank, particularly when injecting liquidity, suggesting that those banks facing a liquidity shortfall can meet a sizeable fraction of their demand from the central bank. This reduced interbank lending then manifests itself in a weaker core-periphery structure. However, the density of the interbank lending network is not affected significantly as on the one hand less interbank lending occurs overall but on the other hand less banks are active in the interbank market. Given that inactive banks are not included in our network analysis, the density remains approximately stable. Overall, differences between the case of banks anticipating the subsequent interbank market and not anticipating it are minimal, thus suggesting that such an anticipation is not important to the interbank structure.

Our observations suggest that monetary policy decisions to inject or extract liquidity affects most strongly banks in the periphery, i. e. mostly smaller banks. Hence, while central bank operations have the desired effect on the interbank market, its effect varies between banks, depending on their position in the interbank market. This has potential implications for central banks as differences in the change of costs for funds can have distributional effects and might well affect the lending these banks do with clients of periphery banks more affected than those of core banks.

Furthermore, the amount of interbank lending and borrowing reduces, potentially affecting the liquidity of the interbank market. By looking in more detail at the borrowing and lending in interbank markets in table 4 we are able to shed some further light on this aspect. We see clearly that interbank borrowing and lending reduces mainly for core banks, with periphery banks actually experiencing an increase in interbank borrowing for liquidity injections and interbank lending for liquidity extractions. Hence with central bank interventions the borrowing and lending of core banks reduces while periphery banks will increase their exposure to the interbank market to meet their liquidity requirements.

Interestingly, core banks will see a reduced return on equity with liquidity injection. With liquidity extraction, their returns remains more or less the same when not anticipating interbank market and are slightly increased when anticipating so. However, periphery banks increase their return on equity in all cases. Both participation in the central bank operation and the interbank market affect banks' return on equity. For the former, we see that periphery banks are more engaging compared to core banks, especially when anticipating the existence of the subsequent interbank

This table shows the medians of key network properties of the interbank lending network. Average rate denotes the median of all interbank rates, CC rate the median interbank rate between core banks, CP rate lending rate from core banks to periphery banks, PC the lending rate from periphery banks to core banks, PP rate the lending rate between periphery banks, Error score denotes the quality of core-periphery network with 0 being a perfect core-periphery structure and 1 showing no such structure at all, Core denotes the fraction of banks in the core, Density the fraction of the possible links between banks that actually exist, and IB lending the total lending in the interbank market. We compare 5 market types, Injection not anticipating IB corresponds to a situation where the central bank injects liquidity but banks do not consider the existence of the interbank market when bidding for central bunk funds, in Injection anticipating IB they do so; likewise we make this distinction in cases where the central bank extracts liquidity and finally on IB only we compare this to a market in which no central bank exists but only the interbank market.

	Average rate	CC rate	CP rate	PC rate	PP rate	Error score	Core	Density	IB lending
Injection not anticipating IB	0.2218	0.2208	0.2423	0.2054	0.2200	0.2463	0.0096	0.0060	411.1864
Injection anticipating IB	0.2295	0.2293	0.2540	0.2122	0.2267	0.2663	0.0099	0.0067	319.3477
Extraction not anticipating IB	0.3520	0.3155	0.4291	0.2832	0.3086	0.1349	0.0090	0.0051	591.8092
Extraction anticipating IB	0.3702	0.3217	0.4631	0.2857	0.3144	0.1241	0.0093	0.0050	658.7691
IB only	0.3270	0.3081	0.4056	0.2553	0.2929	0.1439	0.0105	0.0059	852.3566

Table 3: Medians of interbank market characteristics

market. This is still the case although not as evident when they are not anticipating the interbank market, because auction allocations are then concentrated among much smaller group of banks and not reflected in medians. Even though the amounts involved are small, it can actually increase returns of some periphery banks compared to the case in which no central bank is present. The core banks are much less reliant on central bank funds, their change in returns can be explained by interbank lending/borrowing rates as discussed below.

We see that core banks are borrowing at a lower rate than they are lending with or without the presence of central bank, while for periphery banks this is reversed. The difference between the borrowing rate and lending rate is reduced for both core and periphery banks in liquidity injection and remain roughly the same in liquidity extraction. This means the profit core banks can get from borrowing at lower rate than lending is reduced in liquidity injection and therefore have lower returns. Similarly, periphery banks also benefit from lending/borrowing at a lower/higher rate for liquidity injection and see an increase in returns. And although such benefits are not as pronounced in liquidity extraction, lending to the central bank still increases their returns.

From these statistics we can clearly see that central bank interventions through the injection or extraction of liquidity affects banks differently, depending on their position in the network. With funding costs and liquidity affecting the lending behaviour of banks, such an asymmetry can have an profound impact on the type of companies that are able to receive loans. It is reasonable to say that periphery banks will normally be smaller, regionally focused banks that will have a different client base to the usually larger and often globally acting core banks.

While these results suggest the importance of the position of a bank in the network to assess how it is affected by any policies of the central bank, we have also seen that the network structure itself is affected by the conduct of the central bank. In order to assess how the network structure is affected by the significant number of

This table shows the medians of interbank lending and borrowing properties. Borrowing denotes the amount of interbank borrowing of banks (relative to total assets), Lending the amount of interbank borrowing of banks (relative to total assets), Net is the amount of net interbank borrowing, i. e. interbank borrowing less interbank lending of a bank (relative to total assets), CB the amount of CB borrowing of a bank (relative to total assets), Leverage denotes the leverage ratio of a bank after the interbank market, relative to the initial state, Return its return on equity and finally Borrow rate is the interest rate at which a bank borrows from other banks in the interbank market less the risk-free rate while Lend rate the interest rate at which it lends to other banks in the interbank market less the risk-free rate. We compare 5 market types, Injection not anticipating IB corresponds to a situation where the central bank injects liquidity but banks do not consider the existence of the interbank market when bidding for central bunk funds, in Injection anticipating IB they do so; likewise we make this distinction in cases where the central bank extracts liquidity and finally on IB only we compare this to a market in which no central bank exists but only the interbank market. Core indicates banks in the core and Periphery those in the periphery of the network.

		Borrowing	Lending	Net	CB	Leverage	Return	Borrow rate	Lend rate
Injection not anticipating IB	Core	0.0802	0.0788	0.0063	0.0000	1.0872	0.6766	0.1351	0.1800
	Periphery	0.0071	0.0000	0.0070	0.0000	1.0071	0.5504	0.1653	0.1420
Injection anticipating IB	Core	0.0800	0.0786	0.0076	0.0004	1.0869	0.6883	0.1326	0.1674
	Periphery	0.0050	0.0000	0.0049	0.0030	1.0050	0.5716	0.1677	0.1481
Extraction not anticipating IB	Core	0.0933	0.0893	0.0064	0.0000	1.1029	0.7496	0.2025	0.2888
	Periphery	0.0000	0.0076	−0.0075	0.0000	1.0000	0.5637	0.3041	0.2142
Extraction anticipating IB	Core	0.1140	0.1140	0.0069	−0.0003	1.1286	0.8210	0.2139	0.3026
	Periphery	0.0000	0.0076	−0.0074	−0.0037	1.0000	0.5466	0.3212	0.2147
IB only	Core	0.1430	0.1402	0.0097		1.1669	0.7717	0.1701	0.2656
	Periphery	0.0007	0.0022	−0.0010		1.0007	0.5165	0.2828	0.1881

Table 4: Medians of bank properties in the interbank market

independent variables in our model, we have conducted a number of regressions in tables 5-8 that use the log changes in the network from the situation in which only the interbank market existed as the dependent variable. Firstly we note that the results are highly consistent across the four different market types (liquidity extraction and injection with and without anticipating the interbank market) and thus can be analyzed jointly.

How much the network structure is affected by the fact that a central bank injects or extracts liquidity is mainly determined by the amount of liquidity the central bank seeks to inject or extract (Q^{CB}), the preferences for liquidity (θ), the size of the interbank trading (Q^{IB}), the bank leverage (λ) and in some instances also the size of the liquidity shock ($\frac{\overline{\rho}+\underline{\rho}}{2}$) and its variability ($\overline{\rho} - \underline{\rho}$). In the case of liquidity injection, a larger central bank intervention weakens the core-periphery structure of the interbank network and also reduces the number of banks participating in the interbank network itself. This arises from the fact that with the additional liquidity the banks have less needs to seek such funds in the interbank market, thus reducing their participation and any borrowing and lending gets more equally spread out between banks often trading excess liquidity. The same effect can be observed if banks have a stronger preference for liquidity over returns.

Interestingly, for liquidity extractions this effect is reversed, although the effect here is much smaller. Due to banks' preference for liquidity the lower liquidity in the banking system after the central bank operation will see banks attempting to secure additional funds in the interbank market. Banks in the core are best placed to offer terms that are favourable to other banks due to their increased leverage and this reinforces the core-periphery structure by having banks link to them.

A larger amount of interbank loans in each transaction (Q^{IB}) strengthens the core-periphery structure as larger interbank loans will result in less transactions for the same total amount of borrowing and lending. These fewer transactions will then be more concentrated with the larger banks in the core and thus lead to less transactions

This table shows the parameter estimate of on OLS regression of the indicated independent variables on the dependent variables, that are all defined as log differences of these variables between the variable in case of liquidity injection by the central bank without anticipating the existence of an interbank market and the case of an interbank market only. Component size denotes the size of the largest component in the network, the Core ratio denotes the fraction of banks in the core, the Error score denotes the quality of core-periphery network with 0 being a perfect core-periphery structure and 1 showing no such structure at all, the Density the fraction of the possible links between banks that actually exist and r^{CB} the rate at which the central bank lends to banks. The number in brackets denotes their t-values and *** indicates statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	Component size	Core ratio	Error score	Density	r^{CB}
CONSTANT	−1.7121***	0.7598***	0.7982***	0.8689***	−2.0635***
	(−10.32)	(4.07)	(5.65)	(4.93)	(−21.23)
$\log Q^{CB}$	−0.9675***	0.3502***	0.6716***	0.4465***	−0.4959***
	(−31.11)	(10.02)	(25.33)	(13.50)	(−27.07)
θ	−2.1025***	2.5175***	1.4960***	2.1619***	31.4174***
	(−3.44)	(3.67)	(2.87)	(3.3297)	(86.43)
$\log Q^{IB}$	−0.2068***	0.3574***	−0.0231*	0.3561***	
	(−13.38)	(20.57)	(−1.75)	(21.66)	
$\log N$	0.9016***	−0.3329***	−0.6210***	−0.3995***	0.5023***
	(24.82)	(−8.1511)	(−20.05)	(−10.34)	(23.47)
$\log \lambda$	0.2990***	−0.1988***	−0.1139***	−0.2194***	−0.5742***
	(13.49)	(−7.9772)	(−6.03)	(−9.30)	(−43.82)
$\frac{\overline{\rho}+\underline{\rho}}{2}$	−1.1180	1.2469	−2.0202*	1.6954	−12.7845***
	(−0.86)	(0.85)	(−1.81)	(1.22)	(−16.43)
$\overline{\rho}-\underline{\rho}$	7.7649***	−4.5430***	−2.5224***	−5.7299***	3.8852***
	(10.86)	(−5.65)	(−4.14)	(−7.53)	(9.22)
r^f	−0.1516	−0.2379	−0.2696	−0.1195	0.0136
	(−0.19)	(−0.26)	(−0.39)	(−0.14)	(0.03)
r^D	−0.1413	0.8716	0.1463	0.4482	−2.6807***
	(−0.19)	(1.02)	(0.23)	(0.55)	(−5.91)
r^C	0.3099	−0.2083	−0.1927	−0.1809	3.1764***
	(0.93)	(−0.56)	(−0.68)	(−0.51)	(16.03)
$\overline{Q}$	−0.0188	−0.0052	0.0575	0.0357	−0.0163
	(−0.25)	(−0.06)	(0.91)	(0.45)	(−0.37)
Observations	2957	2957	2957	2957	2957
R^2	0.2930	0.1468	0.2096	0.1761	0.8286

Table 5: Liquidity injection with separated market

between banks in the periphery. Banks with a higher leverage will have a stronger emphasis on profitability due to the impact on return on equity and thus encourage borrowing and lending from them, resulting in a core-periphery structure.

With more banks it is easier to sustain a core-periphery structure and a larger core

This table shows the parameter estimate of on OLS regression of the indicated independent variables on the dependent variables, that are all defined as log differences of these variables between the variable in case of liquidity injection by the central bank with anticipating the existence of an interbank market and the case of an interbank market only. Component size denotes the size of the largest component in the network, the Core ratio denotes the fraction of banks in the core, the Error score denotes the quality of core-periphery network with 0 being a perfect core-periphery structure and 1 showing no such structure at all, the Density the fraction of the possible links between banks that actually exist and r^{CB} the rate at which the central bank lends to banks. The number in brackets denotes their t-values and *** indicates statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	Component size	Core ratio	Error score	Density	r^{CB}
CONSTANT	−1.1113***	0.6532***	0.2401	0.6894***	−1.9990***
	(−5.29)	(2.85)	(1.61)	(3.28)	(−14.81)
$\log Q^{CB}$	−0.4574***	0.0857***	0.2849***	0.1632***	−0.1291***
	(−21.22)	(3.65)	(18.58)	(7.56)	(−9.24)
θ	−1.0433	0.7631	−1.0049*	1.4230*	32.0956***
	(−1.25)	(0.84)	(−1.70)	(1.71)	(59.05)
$\log Q^{IB}$	−0.1350***	0.3263***	−0.0746***	0.3541***	
	(−6.76)	(14.98)	(−5.25)	(17.70)	
$\log N$	0.4054***	−0.1187***	−0.2282***	−0.1452***	0.1642***
	(11.96)	(−3.21)	(−9.46)	(−4.28)	(7.46)
$\log \lambda$	0.1682***	−0.1250***	−0.0123	−0.1536***	−0.6367***
	(6.08)	(−4.15)	(−0.63)	(−5.55)	(−35.40)
$\frac{\overline{\rho}+\underline{\rho}}{2}$	−6.4465***	6.2706***	−0.8292	6.2720***	−12.6672***
	(−3.90)	(3.48)	(−0.71)	(3.79)	(−11.74)
$\overline{\rho}-\underline{\rho}$	6.8850***	−5.0466***	0.4267	−6.1587***	0.7637
	(7.11)	(−4.78)	(0.62)	(−6.35)	(1.22)
r^f	−0.4131	0.2727	0.5891	0.0728	0.2959
	(−0.38)	(0.23)	(0.76)	(0.07)	(0.42)
r^D	−1.4853	1.2928	0.2329	1.6686	−2.1689***
	(−1.43)	(1.14)	(0.32)	(1.61)	(−3.20)
r^C	0.4367	−0.5181	−0.0970	−0.3802	3.2046***
	(0.96)	(−1.04)	(−0.30)	(−0.83)	(10.76)
$\overline{Q}$	0.1065	−0.0773	−0.0544	−0.0564	−0.0308
	(1.068)	(−0.71)	(−0.77)	(−0.57)	(−0.47)
Observations	1639	1639	1639	1639	1639
R^2	0.2366	0.1315	0.2346	0.1856	0.7926

Table 6: Liquidity injection with interdependent market

as generally more trading will occur, allowing for this property to emerge. More diversity in the liquidity shocks banks face ($\overline{\rho}-\underline{\rho}$) strengthens the core-periphery structure while ensuring more banks participate. This arises as more diversity of liquidity needs increases the need and the ability to offset any imbalances banks have in their liquidity positions. If the mean of the liquidity shock increases this

This table shows the parameter estimate of on OLS regression of the indicated independent variables on the dependent variables, that are all defined as log differences of these variables between the variable in case of liquidity extraction by the central bank without anticipating the existence of an interbank market and the case of an interbank market only. Component size denotes the size of the largest component in the network, the Core ratio denotes the fraction of banks in the core, the Error score denotes the quality of core-periphery network with 0 being a perfect core-periphery structure and 1 showing no such structure at all, the Density the fraction of the possible links between banks that actually exist and r^{CB} the rate at which the banks deposit funds at the central bank. The number in brackets denotes their t-values and *** indicates statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	Component size	Core ratio	Error score	Density	r^{CB}
CONSTANT	−0.4712***	0.1522*	0.1617	0.1096*	−1.2496***
	(−11.84)	(1.70)	(1.48)	(1.96)	(−13.80)
$\log Q^{CB}$	−0.0393***	−0.0635***	−0.0246***	−0.0673***	0.1057***
	(−13.69)	(−9.82)	(−3.11)	(−16.66)	(16.08)
θ	0.0547	−1.1632***	−0.4914	0.0162	31.8212***
	(0.35)	(−3.34)	(−1.15)	(0.07)	(89.68)
$\log Q^{IB}$	−0.0590***	0.0718***	0.0226**	0.0800***	
	(−14.59)	(7.89)	(2.03)	(14.08)	
$\log N$	0.0479***	0.0199	−0.0056	0.0401***	−0.0931***
	(8.34)	(1.54)	(−0.36)	(4.98)	(−7.09)
$\log \lambda$	0.0217***	0.0058	0.0214	−0.0098	−0.7112***
	(4.12)	(0.49)	(1.48)	(−1.33)	(−59.17)
$\frac{\overline{\rho}+\underline{\rho}}{2}$	3.3306***	1.5224**	−0.6782	0.8043*	−15.9242***
	(10.58)	(2.15)	(−0.78)	(1.82)	(−22.11)
$\overline{\rho}-\underline{\rho}$	−0.4736**	−1.5931***	−0.3223	−1.7308***	1.3289***
	(−2.52)	(−3.78)	(−0.62)	(−6.57)	(3.10)
r^f	−0.1169	0.6372	−0.1005	0.2800	0.3807
	(−0.58)	(1.40)	(−0.18)	(0.99)	(0.82)
r^D	0.0284	0.0157	0.5748	−0.0617	−2.8166***
	(0.15)	(0.04)	(1.08)	(−0.23)	(−6.35)
r^C	−0.0173	−0.0852	−0.2034	0.0376	3.1769***
	(−0.20)	(−0.45)	(−0.87)	(0.32)	(16.36)
$\overline{Q}$	−0.0140	−0.0119	−0.0236	0.0194	−0.0204
	(−0.75)	(−0.28)	(−0.46)	(0.73)	(−0.47)
Observations	3623	3623	3623	3623	3623
R^2	0.16	0.0718	0.0117	0.1955	0.7935

Table 7: Liquidity extraction with separated market

seems to have very limited effect on the structure of the network, mainly increasing the size of the core and due to the excess liquidity more transactions are required to offset them between banks.

It is note worthy that the overall fit of the regressions reported in tables 5-8, as measured by R^2, is significantly lower for liquidity extraction. The origin of this observation can be found in the fact that for large liquidity extractions the preferences of the banks for liquidity become overwhelming, which leads to non-linearities which these regressions do not allow to incorporate. [2]

Finally we have also considered the determinants of the rate the central bank applies to their lending or deposits, comprising the final column in tables 5-8. Not surprisingly a larger amount of central bank intervention reduces (increases) the interest rate for liquidity injection (extraction). Also intuitively understandable is that banks having a stronger preference for liquidity increases the interest rate. While this result is not surprising, it is worth mentioning explicitly as it shows how preferences of banks, as they vary in light of adverse conditions, might affect funding costs directly and thus at least partially offset the policy decisions of central banks. A larger number of banks competing for a given amount of central bank funds will obviously increase (decrease) the interest rate in the case of liquidity injection (extraction).

If the liquidity shock is on average more positive, this means more liquidity is in the banking system, thus reducing the interest rates due to the lower demand for liquidity. A larger variability of liquidity will, however, increase the interest rates. Here the variability means that more banks face large imbalances and thus are keen to offset these. Those facing a liquidity shortage will demand larger funds from the central bank in the case of a liquidity injection while those facing larger excess liquidity will stay out of this market, thus increasing this interest rate. In the case of liquidity extraction the missing demand for central bank deposits by banks with liquidity shortages will similarly drive up interest rates.

Banking systems with a higher leverage face lower interest rates. Due to the higher leverage the impact of central bank funds on the liquidity ratio is smaller, thus making them less important to the bank and they are demanding less central bank

[2] We note that the R^2 for r^{CB} is very high. As all explanatory variables are chosen exogenously we can discard the problem of endogeneity and in Appendix B show that our results are robust for multicollineraity.

This table shows the parameter estimate of on OLS regression of the indicated independent variables on the dependent variables, that are all defined as log differences of these variables between the variable in case of liquidity extraction by the central bank with anticipating the existence of an interbank market and the case of an interbank market only. Component size denotes the size of the largest component in the network, the Core ratio denotes the fraction of banks in the core, the Error score denotes the quality of core-periphery network with 0 being a perfect core-periphery structure and 1 showing no such structure at all, the Density the fraction of the possible links between banks that actually exist and r^{CB} the rate at which banks deposit funds at the central bank. The number in brackets denotes their t-values and *** indicates statistical significance at the 1% level, ** at the 5% level, and * at the 10% level.

	Component size	Core ratio	Error score	Density	r^{CB}
CONSTANT	−0.5524***	0.2312*	0.1283	0.2571***	−0.7208***
	(−9.79)	(1.68)	(0.65)	(3.35)	(−5.51)
$\log Q^{CB}$	−0.0733***	−0.0764***	−0.1141***	−0.0840***	0.0917***
	(−11.16)	(−4.77)	(−5.00)	(−9.42)	(6.03)
θ	−0.2880	−0.7924	−2.2459***	0.1360	31.4471***
	(−1.35)	(−1.52)	(−3.03)	(0.47)	(63.60)
$\log Q^{IB}$	−0.0615***	0.0858***	−0.0847***	0.1126***	
	(−10.56)	(6.04)	(−4.18)	(14.21)	
$\log N$	0.0754***	0.0493**	0.0374	0.0673***	−0.1201***
	(7.92)	(2.12)	(1.13)	(5.19)	(−5.45)
$\log \lambda$	0.0399***	−0.0167	0.0497*	−0.0276***	−0.7357***
	(5.29)	(−0.90)	(1.89)	(−2.69)	(−42.10)
$\frac{\overline{\rho}+\underline{\rho}}{2}$	3.4400***	−0.1671	3.9430**	0.9526	−17.7083***
	(6.72)	(−0.13)	(2.21)	(1.37)	(−14.93)
$\overline{\rho}-\underline{\rho}$	−0.2146	−0.6882	−1.1652	−2.1675***	2.5934***
	(−0.66)	(−0.86)	(−1.02)	(−4.87)	(3.42)
r^f	−0.0334	−0.3332	−1.2677	−0.0160	0.1506
	(−0.12)	(−0.48)	(−1.29)	(−0.04)	(0.23)
r^D	0.0035	0.9306	1.3341	0.0063	−3.0885***
	(0.01)	(1.44)	(1.45)	(0.02)	(−5.034)
r^C	0.1907	−0.0860	−0.4982	−0.2230	2.8889***
	(1.63)	(−0.30)	(−1.23)	(−1.40)	(10.68)
$\overline{Q}$	−0.0343	−0.1076*	0.0058	0.0009	0.0735
	(−1.32)	(−1.69)	(0.06)	(0.03)	(1.22)
Observations	1571	1571	1571	1571	1571
R^2	0.24	0.0608	0.0561	0.2696	0.8168

Table 8: Liquidity extraction with interdependent market

funds, reducing the interest rate. Also, a higher leverage tends to be associated with a lower return prior to the commencement of the central bank operation, which makes a bank value liquidity less. The observation that higher deposit rates reduce interest rates and higher lending rates increase it arises from their respective effects on the returns of the banks. A higher deposit rate reduces returns to banks and thus banks are more concerned about this aspect. Consequently they are less willing to pay high interest rates while the exact opposite is the case for high lending rates.

5 Conclusions

We have developed a model of the demand by banks for central bank funds. Banks have a preference for liquidity as well as profitability and as such balance those two needs; this is in contrast to most models developed previously that assume banks are only optimizing profitability. Our models considered fixed rate as well as variable rate tenders and also included the anticipation of a subsequent interbank market. We derived the equilibria in those models and were able to show the equivalence of fixed rate and variable rate tenders in terms of interest rates applied and quantities allocated to banks. We also showed the existence of bid shading and flat bids in variable tender tenders.

Furthermore we assessed the structure of the interbank market and how it changes in the presence of a central bank. While we found that overall changes are relatively small, we were able to establish that bank in different positions in the interbank network were affected differently. It was found that the impact of liquidity injections and extractions were affecting banks in the periphery - usually smaller banks - much more than banks in the core - usually larger banks. In the presence of a central bank, banks in the periphery tend to participate more in the interbank market. This asymmetry in the effect on banks arising from the presence of a central bank might have significant policy implications as it could well affect the lending policies of banks differently and subsequently the supply of loans to the economy.

In this paper we only considered auctions, but open market operations are a significant and often the only way central banks manage the liquidity in the banking system. While many aspects of open market operations will be similar to auctions, it would be worth in future research to investigate such a setting. Furthermore we do not consider what banks actually do with the liquidity they obtain as in many cases banks will re-invest them into loans, in particular if liquidity is provided for longer terms. Such re-investments will naturally affect the rate of return and might alter the incentives for banks.

Another interesting extension is to have interbank trading prior to the auction of central bank funds. This is especially useful if we would like to relax the main assumption adopted in this model that the central bank's monetary policy is exogenous. This allows us to apply the framework of this study to explore other important functions of central banks, for instance the lender of last resort, where the aim of liquidity injection is to relief the shortage of liquidity in the banking system. It is possible to consider one such banking system where banks are under high aggregate deposit withdrawals. When the interbank market opens before central bank operations commence, the interbank trading is likely to be less active due to liquidity shortage while the auction of central bank funds afterwards may supply banks with additional liquidity. Further adjustments to the model may be needed to make it more realistic. For example, the amount of liquidity injection could be dependent on the shortage of funds among banks. Also, banks may form an expectation of liquidity injection during their bilateral trading since they know they are short of cash. However such extensions are beyond the scope of this paper and therefore left for future research.

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A Appendix: Proofs

A.1 Proof of Lemma 1

Based on reservation prices in equations (4) and (5) we easily get $\lim_{Q\to 0} r_i^a(Q_i) = r_i^a(0) = r^f + \frac{\theta_i}{1-\theta_i}\frac{(1+r_i^E)\mathbf{E}_i(\mathbf{D}_i+\mathbf{B}_i-\mathbf{R}_i)}{\mathbf{R}_i(\mathbf{D}_i+\mathbf{B}_i)}$ and $\lim_{Q\to 0} r_i^b(Q_i) = r_i^b(0) = r^f + \frac{\theta_i}{1-\theta_i}\frac{(1+r_i^E)\mathbf{E}_i}{\mathbf{R}_i}$. From this we easily obtain that $r_i^b(0) - r_i^a(0) = \frac{\theta_i}{1-\theta_i}\frac{(1+r_i^E)\mathbf{E}_i}{\mathbf{R}_i}\frac{\mathbf{R}_i}{(\mathbf{D}_i+\mathbf{B}_i)} > 0$.

A.2 Proof of proposition 1

Let us first consider the case of liquidity injection, i. e. $Q_i > 0$. The optimization problem (6) has a unique solution as U_i is a concave function of Q_i whose second derivative is

$$\begin{aligned}\frac{d^2U_i}{dQ_i^2} &= -2\frac{\mathbf{D}_i+\mathbf{B}_i-\mathbf{R}_i}{(\mathbf{D}_i+\mathbf{B}_i+Q_i)^3}U_1 + \left(\frac{\mathbf{D}_i+\mathbf{B}_i-\mathbf{R}_i}{(\mathbf{D}_i+\mathbf{B}_i+Q_i)^2}\right)^2 U_{11} \\ &\quad + \frac{\mathbf{D}_i+\mathbf{B}_i-\mathbf{R}_i}{(\mathbf{D}_i+\mathbf{B}_i+Q_i)^2}\frac{r^f-r^{CB}}{\mathbf{E}_i}(U_{12}+U_{21}) + \left(\frac{r^f-r^{CB}}{\mathbf{E}_i}\right)^2 U_{22} < 0,\end{aligned}$$

where $U_1 = \frac{\partial U_i}{\partial \rho_i} > 0$, $U_{11} = \frac{\partial^2 U_i}{\partial \rho_i^2} < 0$, $U_{12} = U_{21} = \frac{\partial^2 U_i}{\partial \rho_i \partial r_i^E} > 0$, $U_{22} = \frac{\partial^2 U_i}{\partial {r_i^E}^2} < 0$.

Therefore the solution to the problem in equation (6) is either at the boundary if one of the constraints is binding or at the local maximum of U_i. Solving for $\frac{\partial U_i}{\partial Q_i} = 0$ gives the local maximum. The first constraint cannot be binding as insolvency gives zero utility. Also note the last constraint is binding when bank i's valuation for borrowing is already lower than r_f^{CB} at $Q_i = 0$, that is $r_i^a(0) < r_f^{CB}$. Therefore, we can write the solution to problem in equation (6) as,

$$Q_i^f\left(r_f^{CB}\right) = \begin{cases} \min\left\{\overline{Q}_i, -\psi_i + \varphi^{\frac{1}{2}}\right\} & \text{if } \; r^f < r_f^{CB} < r_i^a(0) \\ 0 & \text{if } \; r_i^a(0) \le r_f^{CB} \end{cases}, where$$

$\varphi = \psi_i^2 - (\mathbf{D}_i + \mathbf{B}_i)\mathbf{R}_i\frac{r^{CB}-r_i^a(0)}{r^{CB}-r^f}$.

Secondly, consider liquidity extraction, i. e. $Q_i < 0$. Similarly, this problem has a unique solution because U_i is a concave function of Q_i whose second derivative is

$$\frac{d^2U_i}{dQ_i^2} = \frac{U_{11}}{(\mathbf{D}_i + \mathbf{B}_i)^2} + \frac{U_{12} + U_{21}}{\mathbf{D}_i + \mathbf{B}_i} \frac{r^f - r^{CB}}{\mathbf{E}_i} + \left(\frac{r^f - r^{CB}}{\mathbf{E}_i} \right)^2 U_{22} < 0.$$

The second constraint is binding when bank i's valuation for lending is already higher than r_f^{CB} at $Q_i = 0$, or $r_f^{CB} > r_i^b(0)$. Solving for the local maximum by letting $\frac{dU_i}{dQ_i} = 0$ completes the solution.

$$Q_i^f\left(r_f^{CB}\right) = \begin{cases} 0 & \text{if } \ r^f < r_f^{CB} \leq r_i^b(0) \\ \theta_i \frac{\left(1+r_i^E\right)\mathbf{E}_i}{r^{CB} - r^f} - (1-\theta_i)\,\mathbf{R}_i & \text{if } \ r_i^b(0) < r_f^{CB} \end{cases}.$$

Combining these two results gives us the result shown in the proposition.

Dropping the constraint that $Q_i \leq \overline{Q}_i$ as it does not affect the sign of the derivative of $Q_i^f(r)$ we obtain that

$$\frac{\partial Q_i^f}{\partial r_f^{CB}} = \frac{1}{2}\varphi^{-\frac{1}{2}} \quad \left.(\mathbf{D}_i + \mathbf{B}_i)\,\mathbf{R}_i \frac{r^f - r_i^a(0)}{\left(r_f^{CB} - r^f\right)^2}\right) < 0$$

in the case of liquidity injection and

$$\frac{\partial Q_i^f}{\partial r_f^{CB}} = -\theta_i \frac{\left(1 + r_i^E\right)\mathbf{E}_i}{\left(r_f^{CB} - r^f\right)^2} < 0 \tag{9}$$

in the case of liquidity extraction.

A.3 Proof of Lemma 2

The proof is trivial from inverting the equilibrium bid schedule in proposition 1.

A.4 Proof of Lemma 3

We prove the individual parts in turn:

1. By inserting $Q_i^f = 0$ into the inverse bid schedule given in lemma 2 we instantly see that these are identical to the reservation prices defined in lemma 1.

2. By inserting $Q_i^f = 0$ into the inverse bid schedule given in lemma 2 we instantly see that these are identical to the reservation prices defined in lemma 1.

3. $r_i^a(0) - r_i^b(0) = \frac{\theta_i}{1-\theta_i} \frac{\left(1+r_i^E\right)\mathbf{E}_i}{\mathbf{D}_i+\mathbf{B}_i} > 0$ which in combination with claims 1 and 2 of this lemma completes this proof.

4. Suppose there is a $Q_i^f < 0$ such that $r^{CB}\left(Q_i^f\right) < r_i^b\left(Q_i^f\right)$. As the reservation prices are determined such that upon making a deposit of Q_i^f, the utility level does not change from the situation of not making a deposit. Receiving an amount less than $r_i^b\left(Q_i^f\right)$ will reduce the utility level of bank i, contradicting the requirement that $r^{CB}\left(Q_i^f\right)$ maximizes the utility.

5. Suppose there is a $Q_i^f > 0$ such that $r^{CB}\left(Q_i^f\right) > r_i^a\left(Q_i^f\right)$. As the reservation prices are determined such that upon taking a loan from the central bank of Q_i^f, the utility level does not change from the situation of not taking a loan. Paying an amount more than $r_i^a\left(Q_i^f\right)$ will reduce the utility level of bank i, contradicting the requirement that $r^{CB}\left(Q_i^f\right)$ maximizes the utility.

A.5 Proof of Lemma 4

The marginal prices here are bank i's marginal valuation for liquidity. Thus, for $Q_i > 0$, $\widetilde{r}_i^a = \frac{\partial Q_i r_i^a}{\partial Q_i}$, while for $Q_i < 0$, $\widetilde{r}_i^b = \frac{\partial Q_i r_i^b}{\partial Q_i}$, where r_i^a and r_i^b are the reservation prices given determined in equations (4) and (5).

A.6 Proof of Proposition 2

We prove both claims in this proposition in turn, commencing with the case of *liquidity injection*. Let us consider an arbitrary bank i and denote the equilibrium demand schedule of any bank by $Q_i^v(r)$. If all banks, apart from bank i submit their optimal demand schedules and the total supply of liquidity by the central bank is Q^{CB}, the residual demand schedule this bank faces, considering the constraint on the amount it can bid for, is given by

$$Q_i^c(r) = \min\left\{\bar{Q}_i, Q^{CB} - \sum_{j \neq i} Q_j^v(r)\right\},$$

where we require that $r > r^f$. Assume now that Q_i^c is on the optimal demand curve for bank i at an interest rate r^c. This rate r_c would be the lowest possible rate at which the bank can submit its bid and still obtain the requested amount. Due to discriminatory pricing in variable rate auctions, any bid higher than r^c would result in a utility loss as the bank pays more than it has to. Thus for any rate $r > r^c$ the submitted demand is zero. On the other hand, at a rate $r < r^c$ a bid would not be successful as it is too low, hence what price or amount is submitted becomes irrelevant. Hence the only possible equilibrium would be for a bank to submit a bid at exactly r^c for the quantity it requires at that rate.

In the following we show that in equilibrium a bank will submit a bid schedule as indicated in the proposition.

If $r_i^a(0) \leq \widetilde{r}$ the reservation price of not submitting a bid, or equivalently a bid of zero, is optimal as exceeding your reservation price will result in a loss of utility.

In all other cases we now show that alternative points on the residual demand schedule give the bank a lower utility and can thus not be an equilibrium. Let us now consider another equilibrium $\widehat{r} \neq \widetilde{r}$. If we have that $r_i^a(0) > \widehat{r} > \widetilde{r}$, we find that $Q_i^c = \overline{Q}_i$ as can be easily seen by inserting the expressions for Q_j^v into Q_i^c above.

In the case of $\widehat{r} \leq \widetilde{r}$ we compare $\widetilde{Q}_i(\widehat{r})$ and $Q_i^c(\widehat{r})$. By construction $\widetilde{Q}_i(\widehat{r})$ gives the same utility level at $\widehat{r}$ as $Q_i^f(\widetilde{r})$ at $\widetilde{r}$, i. e. it lies on the same indifference curve as the optimal demand schedule. If $\widetilde{Q}_i(\widehat{r}) \geq Q_i^c(\widehat{r})$ then Q_i^c would give the bank less cash than $\widetilde{Q}_i$ at the same price; given that banks prefer more cash, this would lead to a lower utility level and would thus not be optimal.

We now show that $\widetilde{Q}_i(\widehat{r}) \geq Q_i^c(\widehat{r})$ as follows:

$$
\begin{aligned}
\widetilde{Q}_i(\widehat{r}) - Q_i^c(\widehat{r}) &= \widetilde{Q}_i(\widehat{r}) - \min\left\{\bar{Q}_i, Q^{CB} - \sum_{i\neq j} Q_j^v(\widehat{r})\right\} \\
&= \max\left\{\widetilde{Q}_i(\widehat{r}) - Q_i, \widetilde{Q}_i(\widehat{r}) - Q^{CB} + \sum_{j\neq i} Q_j^v(\widehat{r})\right\} \\
&= \max\Big\{\widetilde{Q}_i(\widehat{r}) - \bar{Q}_i, \widetilde{Q}_i(\widehat{r}) - Q^{CB} \\
&\quad + \sum_{j\neq i} \min\left\{\bar{Q}_j, Q_j^f(\widetilde{r}) + \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right)\right\}\Big\} \\
&= \max\Big\{\widetilde{Q}_i(\widehat{r}) - \bar{Q}_i, \widetilde{Q}_i(\widehat{r}) - Q^{CB} \\
&\quad + \sum_{j\neq i} Q_j^f(\widetilde{r}) + \sum_{j\neq i} \min\left\{\bar{Q}_j - Q_j^f(\widetilde{r}), + \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right)\right\}\Big\} \\
&= \max\Big\{\widetilde{Q}_i(\widehat{r}) - \bar{Q}_i, \widetilde{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) \\
&\quad + \sum_{j\neq i} \min\left\{\bar{Q}_j - Q_j^f(\widetilde{r}), \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right)\right\}\Big\} \\
&= \widetilde{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) + \sum_{j\neq i} \min\left\{\bar{Q}_j - Q_j^f(\widetilde{r}), \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right)\right\},
\end{aligned}
$$

where the last step is obtained as $Q_i^f(\widetilde{r}) \leq \bar{Q}_i$ and $\bar{Q}_j - Q_j^f(\widetilde{r})$ and $Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})$ are always nonnegative. If there exist one $j \neq i$, such that $\bar{Q}_j - Q_j^f(\widetilde{r}) \geq \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right)$, we have,

$$\widetilde{Q}_i(\widehat{r}) - Q_i^c(\widehat{r}) \geq \widetilde{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) + \max_{k=1,\ldots,N}\left(Q_k^f(\widetilde{r}) - \widetilde{Q}_k(\widehat{r})\right) \geq 0.$$

Otherwise we have

$$
\begin{aligned}
\widetilde{Q}_i(\widehat{r}) - Q_i^c(\widehat{r}) &= \widetilde{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) + \sum_{j\neq i}\left(\bar{Q}_j - Q_j^f(\widetilde{r})\right) \\
&= \widetilde{Q}_i(\widehat{r}) + \sum_{j\neq i} \bar{Q}_j - Q^{CB} \\
&= \widetilde{Q}_i(\widehat{r}) + (N-1)\bar{Q}_i - Q^{CB} \\
&\geq 0.
\end{aligned}
$$

In the case of *liquidity extraction* the same steps are followed as above. The possible demand by bank i given the demand by all other banks is determined as

$$Q_i^c(r) = \max\left\{-\mathbf{R}_i, Q^{CB} - \sum_{j\neq i} Q_j^v(r)\right\},$$

where we take into account that banks cannot deposit more than their cash reserves and require that $r > r^f$. With the same arguments made before, for any $r < r^c$, the optimal interest rate, the bank does not receive sufficient interest on their deposits with the central bank and thus will bid an amount of zero. Furthermore if $r_i^b(0) \geq \widetilde{r}$ the reservation price is too high for the bank to bid for depositing cash with the central bank, and thus will also bid an amount of zero.

For the case of $r_i^b(0) < \widehat{r} < \widetilde{r}$ we can easily show that $Q_i^c(r) = \max\left(-\mathbf{R}_i, Q^{CB}\right)$ by inserting for $Q_j^v(r)$. In the case that $\widehat{r} \geq \widetilde{r}$ we follow the same arguments as in the case of liquidity injection and need to show that $\overline{Q}_i(\widehat{r}) \leq Q_i^c(\widehat{r})$. We obtain

$$\begin{aligned}
\overline{Q}_i(\widehat{r}) - Q_i^c(\widehat{r}) &= Q_i(\widehat{r}) - \max\left\{-\mathbf{R}_i, Q^{CB} - \sum_{j\neq i} Q_j^v(\widehat{r})\right\} \\
&= \min\left\{\overline{Q}_i(\widehat{r}) + \mathbf{R}_i, \overline{Q}_i(\widehat{r}) - Q^{CB} + \sum_{j\neq i} Q_j^v(\widehat{r})\right\} \\
&= \min\left\{\overline{Q}_i(\widehat{r}) + \mathbf{R}_i, \overline{Q}_i(\widehat{r}) - Q^{CB} + \sum_{j\neq i}\left(Q_j^f(\widetilde{r}) - \max_{k=1,\ldots,N}\left(\overline{Q}_k(\widehat{r}) - Q_k^f(\widetilde{r})\right)\right)\right\} \\
&= \min\left\{\overline{Q}_i(\widehat{r}) + \mathbf{R}_i, \overline{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) - (N-1)\left(\max_{k=1,\ldots,N}\left(\overline{Q}_k(\widehat{r}) - Q_k^f(\widetilde{r})\right)\right)\right\} \\
&= \overline{Q}_i(\widehat{r}) - Q_i^f(\widetilde{r}) - (N-1)\left(\max_{k=1,\ldots,N}\left(\overline{Q}_k(\widehat{r}) - Q_k^f(\widetilde{r})\right)\right) \\
&\leq 0
\end{aligned}$$

The penultimate step arises as $Q_i^f(\widetilde{r}) \geq -\mathbf{R}_i$ and $\overline{Q}_k(\widehat{r}) \geq Q_k^f(\widetilde{r})$.

A.7 Proof of Lemma 5

This lemma follows from the definition of the inverse bid schedule, $r_i^v(Q_i) = \inf\left\{r > r^f | Q_i^v(r) \leq Q_i\right\}$. It is also obvious that $r_i^v(Q_i)$ is a non-increasing function of Q_i. To show this, we only need to show that $R^{-1}(Q_i)$ is a non-increasing function of Q_i. This is true as when r increases, both $\min\left\{\bar{Q}_i, Q_i^f(\widetilde{r}) + \max_{j=1,\ldots,N}\left(Q_j^f(\widetilde{r}) - \widetilde{Q}_j(r)\right)\right\}$ and $Q_i^f\left(\widetilde{\widetilde{r}}\right) -$

$\max_{j=1,\ldots,N}\left(\widetilde{\widetilde{Q}}_j(r)-Q_j^f\left(\widetilde{\widetilde{r}}\right)\right)$ are non-increasing as can easily be seen.

A.8 Proof of proposition 3

We need to show the tuple $\left(r_f^{CB}, Q_1^f, \ldots, Q_N^f\right)$ also clears the market when bid schedules are as described in proposition 2. First consider the case of $Q_i^f \geq 0$, when the amount of operation is $Q^{CB} = \sum_{i=1}^N Q_i^f = \sum_{i=1}^N \max\left(0, Q_i^f\left(r_f^{CB}\right)\right)$, obviously $r_f^{CB} \in \left\{r \geq r^f \mid \sum_i^N \max\left(0, Q_i^f(r)\right) = Q^{CB}\right\}$. Note that $Q_i^f(r)$ is strictly decreasing before reaching limit $\bar{Q}_i$. Therefore, $\sum_i^N \max\left(0, Q_i^f(r)\right)$ is strictly decreasing as $0 < Q^{CB} < (N-1)\bar{Q}_i$ and has thus a unique solution. Therefore, $\widetilde{r}$ defined in proposition 2 equals r_f^{CB}, since $Q_i^v(r^*) = Q_i^f(r^*)$ (if $\widetilde{r} > r_i^a(0)$ this also holds as both are zero), what remains to be shown is $r_v^{CB} = \widetilde{r}$. This is obvious as $\sum_{i=1}^N Q_i^v(\widetilde{r}) = Q^{CB}$ clears the market while any $r < \widetilde{r}$ cannot.

The proof for $Q_i^f \leq 0$ follows exactly the same process.

A.9 Proof of Proposition 4

It is easy to verify that $\left(\widehat{r}^{IB}, \lambda Q_1^I\left(\widehat{r}^{IB}\right), \ldots, \lambda Q_1^I\left(\widehat{r}^{IB}\right)\right)$ clears the market. We show here that a bank i cannot gain higher expected utility after the interbank market by deviating from the bid schedule proposed here. As shown in proposition 2, it is optimal for any bank i to pay no more than the clearing rate of the primary market, so in the following we only consider bid schedules where a bank demands zero if the interest rate charged is larger than expected clearing rate in the interbank market. We show the case for liquidity injection here as for liquidity extraction the argument can be made in exactly the same way.

Let us consider bank i having any alternative bid schedule where it bids $Q_i > 0$ at a rate r_i^{CB} and $Q_i = 0$ at some rate $r > r_i^{CB}$. Firstly, if $r_i^{CB} < \widehat{r}^{IB}$, bank i does not participate in the primary market but only the interbank market. This does not change the clearing rate as the reduced allocation to bank i is compensated by increased allocation to other banks. Consequently, in the interbank market, bank i demands more funds while other banks are expected to demand less due to their increased allocation by the central bank, hence the aggregate amount is unchanged and so is the expected interbank rate. Here bank i only shifts part of its demand from central bank funds to the interbank market and its expected overall utility increase is the same; hence it is not better off.

Secondly, if $r_i^{CB} = \widehat{r}^{IB}$, but $Q_i^f\left(\widehat{r}^{IB}\right) \geq Q_i \neq Q_i^I\left(\widehat{r}^{IB}\right)$, this results in bank i borrowing less from the central bank as the rate is less favourable and this has a similar effect as in the case where $r_i^{CB} < \widehat{r}^{IB}$. If $r_i^{CB} = \widehat{r}^{IB}$, but $Q_i^f\left(\widehat{r}^{IB}\right) < Q_i \neq Q_i^f\left(\widehat{r}^{IB}\right)$, bank i could be worse off. Its allocation could exceed $Q_i^f\left(\widehat{r}^{IB}\right)$ which is the optimal amount that maximize i's utility or by over-reporting its demand bank i also makes the demand for liquidity to be greater in subsequent interbank markets and thus raises the expected interbank rate for all banks, including itself.

Thirdly, if $r_i^{CB} > \widehat{r}^{IB}$ and if $Q_i \geq Q_i^I\left(\widehat{r}^{IB}\right)$ bank i would be strictly worse off because it pays more for liquidity from the central bank as well as the interbank market. The former is obvious and the latter is because over-reporting bank i's demand raises expected interbank rates as discussed above. On the other hand if $Q_i < Q_i^I\left(\widehat{r}^{IB}\right)$, bank i would still be worse off. Suppose in this case, bank i gets an allocation of Q_i^{CB} from the central bank and demands Q_i^{IB} in the interbank market. Obviously, for Q_i^{CB}, bank i pays more than $\widehat{r}^{IB}$ which reduces its utility. For Q_i^{IB}, there is a chance bank i pays less than $\widehat{r}^{IB}$, even so, this is not enough to compensate for i's utility loss from central bank funds. Suppose the opposite is true, that bank i pays in the interbank market $r_i^{IB} < \widehat{r}^{IB}$, keeping its utility the same as before. Consider all combinations of rate and quantity (r, Q) in the interbank market that gives the same utility as originally, i. e. $\left\{(r,Q)|U_i\left(\rho_i(Q), r_i^E(r,Q)\right) = U_i\left(\rho_i\left(Q_i^I\left(\widehat{r}^{IB}\right)\right), r_i^E\left(\widehat{r}^{IB}, Q_i^I\left(\widehat{r}^{IB}\right)\right)\right), \widehat{r}^{IB} > r > r^f\right\}$. If the collateral constraint is not binding for bank i, it has to demand Q_i^{IB} in order to maximize its utility, corresponding to the maximum of r, but $r < widehatr^{IB}$ implies $Q_i^{IB} + Q_i^{CB} \geq Q_i^I\left(\widehat{r}^{IB}\right)$. If the collateral is binding, equality holds here.

Overall, this implies interbank markets cannot clear as banks' aggregate demand must be more than the supply due to a drop in interbank market rate and the fact bank i is also demanding more than or equal as before. Therefore, bank i cannot reach the same level of utility as originally.

B Appendix: Robustness for multicollineraity

In tables 5,6,7,8 the regressions on r^{CB} have quite high R^2. We can rule out the possibility of endogeneity since the independent variables are parameters of our model which are exogenous. We investigate the correlation matrices among independent variables and find that most of them are only weakly correlated. However, we do find three exceptions:

1. r^f and r^D, r^f and r^C are moderately correlated;
2. mean and variance of the liquidity distribution are also strongly correlated;
3. the amount of central bank operation is correlated with the number of total banks.

The first exception is consistent with our setting, as r^D and r^C have to be within reasonable range (not too distant from r^f) so that we can observe central bank operations and subsequent interbank trading. The second exception is expected from the construction and the third exception arises as the operation amount needs to be reasonable compared to the size of the banking system, such that there could be sufficient interbank trading and thus we include it in our sample. In other words, in order to observe sufficient interbank trading, the number of total banks and the amount of operation need to be compatible with each other, this serves as the source for observed correlation. We also consider alternative regression models that exclude r^f, the number of total banks, N, and variance of the liquidity distribution, $\overline{\rho}-\underline{\rho}$, for each of the tables 5, 6, 7, 8. And we find that these changes have limited effects on our regression results (see table 13), particularly the R^2 of each regression is about the same level as the original regression and the signs and magnitudes of coefficients remain unchanged.

Correlation	$\log(Q^{CB})$	θ	$\log(N)$	λ	$\frac{\overline{\rho}+\underline{\rho}}{2}$	$\overline{\rho}-\underline{\rho}$	r^f	r^D	r^C	$\overline{Q}$
$\log(Q^{CB})$	1.0000									
θ	0.0031	1.0000								
$\log(N)$	0.7447	0.0106	1.0000							
λ	0.1617	0.0272	0.0104	1.0000						
$\frac{\overline{\rho}+\underline{\rho}}{2}$	0.3701	−0.0176	−0.0287	0.0213	1.0000					
$\overline{\rho}-\underline{\rho}$	0.3094	−0.0139	−0.0346	0.0201	0.9004	1.0000				
r^f	−0.0147	−0.0132	−0.0130	0.0133	−0.0225	−0.0383	1.0000			
r^D	0.0017	−0.0139	−0.0095	−0.0129	−0.0032	−0.0093	0.5577	1.0000		
r^C	−0.0311	0.1024	−0.0180	−0.0494	−0.0225	−0.0311	0.4865	0.2708	1.0000	
$\overline{Q}$	−0.0356	−0.0063	0.0093	−0.0179	−0.0469	−0.0367	0.0418	0.0326	0.0317	1.0000

Table 9: Correlation matrix of variables in table 5.

Correlation	$\log(Q^{CB})$	θ	$\log(N)$	λ	$\frac{\overline{\rho}+\underline{\rho}}{2}$	$\overline{\rho}-\underline{\rho}$	r^f	r^D	r^C	$\overline{Q}$
$\log(Q^{CB})$	1.0000									
θ	0.0054	1.0000								
$\log(N)$	0.5663	0.0113	1.0000							
λ	0.1228	0.0315	0.0205	1.0000						
$\frac{\overline{\rho}+\underline{\rho}}{2}$	0.3154	−0.0116	−0.0023	0.0189	1.0000					
$\overline{\rho}-\underline{\rho}$	0.3446	−0.0116	−0.0175	0.0267	0.9081	1.0000				
r^f	−0.0037	−0.0462	0.0179	0.0166	−0.0163	−0.0221	1.0000			
r^D	0.0194	−0.0514	0.0224	−0.0287	−0.0233	−0.0106	0.5437	1.0000		
r^C	−0.0069	0.1097	0.0245	0.0142	−0.0154	−0.0286	0.4857	0.2388	1.0000	
$\overline{Q}$	−0.0629	0.0002	0.0076	−0.0246	−0.0278	−0.0262	−0.0004	0.0174	0.0199	1.0000

Table 10: Correlation matrix of variables in table 6.

Correlation	$\log(Q^{CB})$	θ	$\log(N)$	λ	$\frac{\overline{\rho}+\underline{\rho}}{2}$	$\overline{\rho}-\underline{\rho}$	r^f	r^D	r^C	$\overline{Q}$
$\log(Q^{CB})$	1.0000									
θ	−0.0039	1.0000								
$\log(N)$	0.4483	0.0115	1.0000							
λ	0.0692	0.0340	0.0076	1.0000						
$\frac{\overline{\rho}+\underline{\rho}}{2}$	0.2373	−0.0156	−0.0188	−0.0014	1.0000					
$\overline{\rho}-\underline{\rho}$	0.3513	−0.0156	−0.0169	−0.0002	0.9034	1.0000				
r^f	−0.0156	−0.0106	−0.0127	0.0096	−0.0148	−0.0252	1.0000			
r^D	0.0036	−0.0074	−0.0056	−0.0185	−0.0003	−0.0020	0.5514	1.0000		
r^C	−0.0207	0.1104	−0.0105	−0.0478	−0.0191	−0.0247	0.4895	0.2767	1.0000	
$\overline{Q}$	0.0305	−0.0041	0.0129	−0.0122	−0.0108	−0.0008	0.0316	0.0332	0.0273	1.0000

Table 11: Correlation matrix of variables in table 7.

Correlation	$\log(Q^{CB})$	θ	$\log(N)$	λ	$\frac{\overline{\rho}+\underline{\rho}}{2}$	$\overline{\rho}-\underline{\rho}$	r^f	r^D	r^C	$\overline{Q}$
$\log(Q^{CB})$	1.0000									
θ	−0.0346	1.0000								
$\log(N)$	0.6028	0.0119	1.0000							
λ	0.0790	0.0266	−0.0047	1.0000						
$\frac{\overline{\rho}+\underline{\rho}}{2}$	0.1777	−0.0048	−0.0314	0.0033	1.0000					
$\overline{\rho}-\underline{\rho}$	0.3378	−0.0018	−0.0249	0.0078	0.9148	1.0000				
r^f	−0.0690	0.0002	−0.0520	0.0110	−0.0273	−0.0450	1.0000			
r^D	−0.0384	0.0115	−0.0299	0.0090	0.0023	−0.0101	0.5674	1.0000		
r^C	−0.0759	0.1091	−0.0607	−0.1048	−0.0275	−0.0239	0.4903	0.2921	1.0000	
$\overline{Q}$	0.0052	−0.0036	−0.0004	0.0022	−0.0592	−0.0497	0.0698	0.0353	0.0287	1.0000

Table 12: Correlation matrix of variables in table 8.

	(1)	(2)	(3)	(4)
CONSTANT	-0.5420***	-1.2970***	-1.7568***	-1.3064***
	(-6.68)	(-13.37)	(-29.51)	(-13.82)
$\log Q^{CB}$	-0.1431***	-0.0643***	0.0909***	0.0634***
	(-12.87)	(-5.85)	(16.73)	(6.33)
θ	31.6816***	32.1331***	31.7588***	31.3204***
	(79.02)	(58.43)	(88.90)	(62.43)
$\log \lambda$	-0.6726***	-0.6554***	-0.7050***	-0.7272***
	(-49.14)	(-36.31)	(-58.48)	(-41.43)
$\frac{\overline{\rho}+\underline{\rho}}{2}$	-11.5978***	-12.3574***	-13.5551***	-13.3554***
	(-31.00)	(-25.56)	(-43.32)	(-30.00)
r^D	-2.9156***	-2.0360***	-2.6195***	-3.0857***
	(-6.75)	(-3.42)	(-6.74)	(-5.72)
r^C	3.0531***	3.3024***	3.2491***	3.0062***
	(15.41)	(12.19)	(18.34)	(12.02)
$\overline{Q}$	-0.0343	-0.0064	-0.0178	0.0799
	(-1.32)	(-0.10)	(-0.41)	(1.30)
Observations	2957	1639	3623	1571
R^2	0.7902	0.7854	0.7894	0.8091

Table 13: OLS regression against r^{CB} with r^f, $\log(N)$ and $\overline{\rho} - \underline{\rho}$ excluded. (1) is to be compared with table 5, (2) with table 6, (3) with table 7, and (4) with table 8.

Chapter 5

Conclusions

We show in general how interbank lending networks emerge, and what determinants drive their structures and give rise to their properties. Essay 1 has presented a model of interbank lending where risk averse banks seek to maximize expected profits by observing and extracting information from other banks' lending decision. It has provided a link between asymmetric information and interbank market freeze. Essay 2 introduced a framework where banks balance their needs for cash and return by bilateral interbank transactions when facing idiosyncratic liquidity shocks. Realistic results were obtained including emergence of the core-periphery structure and endogenously determined interest rates. Essay 3 based on the interbank market in essay 2 introduced a central bank operation in the form of auction. The monetary policy is found to affect individual banks differently while the general structure of the interbank market is maintained. The study helps to better understand systemic risk which is shown to be dependent on interbank network structures. Thus it is also of importance for the macro prudential regulation which aims to manage such risk. Moreover, monetary policy is shown in the study to impact banks differently. An awareness of the degree of such asymmetry can help to the correct implementation and transmission of monetary policy.

Admittedly, there are some limitations in this study and some can be left for future work. For instance, in essay 1, we consider a three-bank interbank market for simplicity. The limitation in the number of banks restrain us from exploring properties that only emerge in large interbank networks, and from considering alternative assumptions like a partial observation of the interbank network. Another limitation of essay 1 is that our setting for the interbank rollover lending is static, while a more complete analysis should consider rollovers that are made dynamically. For the model in essay 3, we limit the monetary policy in our model to only auctions of central bank funds, while in reality various forms of open market operation are also influential and worth studying. In addition, our model also do not consider banks that can use short-term funding to re-invest into long-term loans. Taking re-investment into consideration will certainly influence banks' profits and include a risk of default, which consequently change banks' bidding and trading behaviours.